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# BATHURST'S ARITHMETIC.

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234.

# THE ELEMENTS

OF

# ARITHMETIC,

EXPRESSLY DESIGNED

FOR THE USE OF SCHOOLS,

BY

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Rochester:
WILLIAM WILDASH.
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# PREFACE.

THE Author having been for some years engaged in Tuition, and having found the want of an Arithmetic in which the Rules were sufficiently plain for children to comprehend, it has been his endeavour in these few pages to comprise all the rules generally taught, and to give such explanations as seemed necessary.

The examples have been made to bear, as much as possible, upon the transactions of every day life, and are given without answers, in order that the Pupil may be made to think a little for himself, the Author being convinced, that after the rules have been properly explained, the less assistance given the better. Indeed it must have occurred to every person engaged in Tuition, that the answers, generally given in treatises upon Arithmetic, are a complete key to the stating of questions in the Rule of Three, and the numerous rules depending upon it.

The introductions to some of the rules, as the Rule of Three, Buying and Selling of Stock, &c. are not intended like the rules themselves, to be committed to memory, but to be read over attentively, so that questions put by the Tutor may be readily answered by the Pupil.

The Author hopes that these introductions, though somewhat prolix, may yet prove useful, his object having been to make an Arithmetic not so much for the Tutor as the Pupil.

C. B.

Rochester, April, 1842.

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# EXPLANATION OF THE SIGNS

## MADE USE OF

## IN THE FOLLOWING WORK.

+ *Plus or more,* the sign of addition, signifies that the two numbers between which it is placed are to be added together; thus 3 + 4 means that 3 is to be added to 4.

— *Minus or less,* the sign of subtraction, signifies that the latter of the two numbers between which it is placed is to be subtracted from the former; thus 4—3 means that 3 is to be subtracted from 4.

× *Multiplied into,* the sign of Multiplication, signifies that the numbers between which it is placed are to be multiplied together; thus 3 × 4 means that 3 is to be multiplied by 4.

÷ *Divided by,* the sign of division, signifies that the former of the two numbers between which it is placed is to be divided by the latter; thus $4 \div 3$ means that 4 is to be divided by 3; and is usually written thus $\frac{4}{3}$.

$=$ *Equal to*, the sign of equality, signifies that the numbers or quantities between which it is placed are equal to each other; thus $3+4=7$ means that the number 3 added to the number 4 is equal to the number 7.

# ARITHMETIC.

ARITHMETIC is the science that treats of the nature and use of numbers.

*Number* is either an unit (1) or a collection of units, as, 2, 3, 7, &c.—where 1, 2, 3, 7 are called *figures*.

*The figures are nine in number*, viz., 1, one ; 2, two ; 3, three ; 4, four ; 5, five ; 6, six ; 7, seven ; 8, eight ; 9, nine. By means of these figures, together with 0, nought or cipher, all numbers may be represented.

Arithmetic consists of two parts, integral and fractional.

*Integral Arithmetic* treats of integers or whole quantities, which are used to express a number of things of the same kind, as, 3 pounds, 7 shillings, 5 miles, &c.

*Fractional Arithmetic* treats of quantities, which are parts of some integer, or whole quantity, as, $\frac{1}{2}$, $\frac{3}{4}$, &c.—where $\frac{1}{2}$ and $\frac{3}{4}$ represent one-half and three-fourths of an integer respectively.

The fundamental rules of Arithmetic are Notation, Numeration, Addition, Subtraction, Multiplication, and Division.

*Notation* is the art of expressing numbers by their proper figures; and teaches us to write down any number, and to have a knowledge of the exact value of every figure composing that number.

*Numeration* is the art of expressing by words the numbers so represented.

The figures in any number besides their own particular value, have another value according to their situation in that number, the figure towards the right hand expresses so many units, and is said to be in the *unit's place,* and of the figures to the left of the unit's place,

the 2nd figure expresses so many tens,
3rd .. .. hundreds,
4th .. .. thousands,
5th .. .. tens of thousands,
6th .. .. hundreds of thousands,
7th .. .. millions,
8th .. .. tens of millions,
9th .. .. hundreds of millions,
&c., &c., &c.,

thus in the number 987654321, the figure to the right hand expresses one unit,

the 2nd figure expresses .. two tens or twenty,
3rd ...... .. three hundreds,
4th ...... .. four thousands,
5th ...... .. fifty thousands,
6th ...... .. six hundred thousands,
7th ...... .. seven millions,
8th ...... .. eighty millions,
9th ...... .. nine hundred millions;

and the number is read nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one.

## EXAMPLES.

Write down in words at length the following numbers:—

27
348
659
3563
33333
10501
23047
167320
735060
13526954
270493756
197059607

Write down in figures the following numbers:—

Forty-nine; three hundred and seventy-six; four hundred; two thousand, six hundred and thirty; seventy-six thousand, four hundred and three; one million, three hundred and two thousand, four hundred and ninety; three hundred and fifty-eight millions, eight hundred and seventy-three thousand and six.

## SIMPLE ADDITION.

SIMPLE ADDITION teaches us to collect several numbers into one sum.

RULE. Place the numbers under each other, so that units may be under units, tens under tens, &c.; then, beginning at the units, add up each column separately, and put down the sum if less than ten; but if the sum is greater than ten, put down the units and carry the tens to the next column, and so on, continuing to the last column, at which set down the total amount.

PROOF. Add together all the numbers except the first, then add their sum to that first number, and if the result is the same as the sum of all the numbers, the work is correct.

### EXAMPLES.

1. Add 23765, 4983, 87539, 4156 together.

$$\begin{array}{r} 23765 \\ 4983 \\ 87539 \\ 4156 \\ \hline 120443 \\ \hline\hline \end{array}$$

EXPLANATION. Here the sum of the 1st column is 23, the units (3) are to be put down, and the tens (2) are to be carried to the next column, and so on.

2. Add 35914, 73647, 39149, 87365, 41637 together.

3. Add 415763, 293496, 375269, 417398, 843275 together.

4. Add 135265, 4175, 369487, 39278, 149204 together.

5. Add 714638, 1347, 84731, 482753, 618279 together.

6. Add 138265, 7048, 36145, 827304, 710487 together.

7. Add 91943, 82796, 394956, 415293, 10475 together.

## SIMPLE SUBTRACTION.

SIMPLE SUBTRACTION teaches us to subtract or deduct a less number from a greater, whereby the difference or remainder is found.

RULE. Place the numbers one under the other, according to the value of their places, as in Addition. Then, beginning at the units, subtract each figure in the lower line from the figure above it, and set down the remainder: if the lower figure be greater than the upper, add *ten* to the upper and then subtract the lower figure from that sum, observing in this case to carry *one*, for the ten that was borrowed, to the next lower figure.

PROOF. To the lesser number add the remainder: if the sum be like the greater number the work is correct.

EXAMPLES.

1. From 376184 take 91837.

```
376184
 91837
------
284347  Remainder
------
376184  Proof.
======
```

2. From 763291843 take 67529376
3. From 476104276 take 317105367
4. From 870476193 take 70483764

## SIMPLE MULTIPLICATION.

SIMPLE MULTIPLICATION teaches us to add the greater of two given numbers as often as there are units in the less; being a short method of performing addition.

### MULTIPLICATION TABLE.

2 times 1 are 2
2 times 2 are 4
2 times 3 are 6
2 times 4 are 8
2 times 5 are 10
2 times 6 are 12
2 times 7 are 14
2 times 8 are 16
2 times 9 are 18
2 times 10 are 20
2 times 11 are 22
2 times 12 are 24

3 times 1 are 3
3 times 2 are 6
3 times 3 are 9
3 times 4 are 12
3 times 5 are 15
3 times 6 are 18
3 times 7 are 21
3 times 8 are 24
3 times 9 are 27
3 times 10 are 30
3 times 11 are 33
3 times 12 are 36

4 times 1 are 4
4 times 2 are 8
4 times 3 are 12
4 times 4 are 16
4 times 5 are 20
4 times 6 are 24
4 times 7 are 28
4 times 8 are 32
4 times 9 are 36
4 times 10 are 40
4 times 11 are 44
4 times 12 are 48

5 times 1 are 5
5 times 2 are 10
5 times 3 are 15
5 times 4 are 20
5 times 5 are 25
5 times 6 are 30
5 times 7 are 35
5 times 8 are 40
5 times 9 are 45
5 times 10 are 50
5 times 11 are 55
5 times 12 are 60

6 times 1 are 6
6 times 2 are 12
6 times 3 are 18
6 times 4 are 24
6 times 5 are 30
6 times 6 are 36
6 times 7 are 42
6 times 8 are 48
6 times 9 are 54
6 times 10 are 60
6 times 11 are 66
6 times 12 are 72

7 times 1 are 7
7 times 2 are 14
7 times 3 are 21
7 times 4 are 28
7 times 5 are 35
7 times 6 are 42
7 times 7 are 49
7 times 8 are 56
7 times 9 are 63
7 times 10 are 70
7 times 11 are 77
7 times 12 are 84

8 times 1 are 8
8 times 2 are 16
8 times 3 are 24
8 times 4 are 32
8 times 5 are 40
8 times 6 are 48
8 times 7 are 56
8 times 8 are 64
8 times 9 are 72
8 times 10 are 80
8 times 11 are 88
8 times 12 are 96

9 times 1 are 9
9 times 2 are 18
9 times 3 are 27
9 times 4 are 36
9 times 5 are 45
9 times 6 are 54
9 times 7 are 63
9 times 8 are 72
9 times 9 are 81
9 times 10 are 90
9 times 11 are 99
9 times 12 are 108

10 times 1 are 10
10 times 2 are 20
10 times 3 are 30
10 times 4 are 40
10 times 5 are 50
10 times 6 are 60
10 times 7 are 70
10 times 8 are 80
10 times 9 are 90
10 times 10 are 100
10 times 11 are 110
10 times 12 are 120

11 times 1 are 11
11 times 2 are 22
11 times 3 are 33
11 times 4 are 44
11 times 5 are 55
11 times 6 are 66
11 times 7 are 77
11 times 8 are 88
11 times 9 are 99
11 times 10 are 110
11 times 11 are 121
11 times 12 are 132

12 times 1 are 12
12 times 2 are 24
12 times 3 are 36
12 times 4 are 48
12 times 5 are 60
12 times 6 are 72
12 times 7 are 84
12 times 8 are 96
12 times 9 are 108
12 times 10 are 120
12 times 11 are 132
12 times 12 are 144

The principal parts of a Multiplication sum are *three*, viz.:—

1. *The Multiplicand,* or number to be multiplied.

2. *The Multiplier,* or number by which you multiply.

3. *The Product,* or number produced by multiplication.

**Rule.** Place the numbers one under the other, according to the value of their places, as in Addition. Multiply each figure in the multiplicand by the first figure in the multiplier, beginning at the unit's place, and set down the product, if less than ten, under each figure so multiplied; but if more than ten, set down the units and carry the tens, taking care to place the first figure in every line under the figure you multiply by; then add these products together for the whole product.

**Proof.** The only correct method of proving a Multiplication sum is by Division, which the beginner, not having learned, cannot practise.

### EXAMPLES.

1. Multiply 7148369 by 7.

| | |
|---:|:---|
| 7148369 | Multiplicand |
| 7 | Multiplier |
| 50038583 | Product. |

2. Multiply 3691047 by 375.

$$\begin{array}{rl} 3691047 & \text{Multiplicand} \\ 375 & \text{Multiplier} \\ \hline 18455235 & \\ 25837329\phantom{0} & \\ 11073141\phantom{00} & \\ \hline 1384142625 & \text{Product.} \\ \hline\hline \end{array}$$

3. Multiply 738269154 by 6,7,8,9,11, and 12.
4. Multiply 147382763 by 87.
5. Multiply 473290475 by 93.
6. Multiply 82900761 by 148.
7. Multiply 307109367 by 7265.
8. Multiply 4716938 by 619478.
9. Multiply 7304975 by 307609.
10. Multiply 5618473 by 9300407.
11. Multiply 3542768 by 3075089.
12. Multiply 71938265 by 71938265.

## SIMPLE DIVISION.

Simple Division teaches us to find how often the lesser of two numbers is contained in the greater.

The principal parts of a Division sum are *three*, viz.

1. *The Dividend*, or number to be divided.

2. *The Divisor*, or number by which you divide.

3. *The Quotient*, or number that shows how often the Divisor is contained in the Dividend.

If the Divisor is not contained a certain number of times exactly in the Dividend, there will be a *fourth* number, called a ***Remainder***, which will always be less than the Divisor.

Simple Division may be divided into two cases :—

1. Short Division, or where the Divisor *does not exceed* 12.

2. Long Division, or where the Divisor *does exceed* 12.

## SHORT DIVISION.

**Rule.** Find how often the divisor is contained in the first or two first figures of the dividend, and set down the quotient under the figure or figures divided, and, if there be a remainder, carry 10 for every unit remaining to the next figure of the dividend, and find how often the divisor is contained in this number, set down the quotient, and proceed as before.

**Proof.** Multiply the quotient by the Divisor, add the remainder, if any, to the product, and the sum will give the dividend.

### Examples.

1. Divide 371596352 by 7.

Divisor 7) 371596352 Dividend
Quotient 53085193 1 Remainder
7
371596352 Proof.

**Explanation.** Here the divisor (7) is contained in the first two figures of the dividend (37) five times with two units (2) remaining, for five-times seven are thirty-five, and thirty-five subtracted from thirty-seven, two remain, carry two tens or 20 to the next figure of the dividend (1), and find how often the divisor (7) is contained in this number (21), and proceed as before.

2. Divide 67193526938 by 4.
3. Divide 7158403276 by 5.
4. Divide 4732691547 by 6.
5. Divide 6320450763 by 7.
6. Divide 1727040735 by 8.
7. Divide 6827507079 by 9.
8. Divide 3070690871 by 11.
9. Divide 14635270918 by 12.

It may here be observed that when the divisor is 10, the quotient will be the same as the dividend, with the exception of the last figure, which will be the remainder.

## LONG DIVISION.

**Rule.** Find how often the *first figure* of the divisor is contained in the first figure or first two figures of the dividend, and place the figure that expresses that number on the right hand of the dividend for the first figure of the quotient; multiply the divisor by that figure, and place the product under as many figures of the dividend as are just necessary; subtract; add to the right hand of the remainder the next figure of the dividend and proceed as before, and so on through all the figures of the dividend.

**Proof.** The same as for Short Division.

**Observations.** 1. If the first figure of the divisor *is contained* in the first figure of the dividend, *as many* places of figures must be taken in the dividend for the first divisor as there are places of figures in the divisor;—but if *not*, then *one more* place must be taken in the dividend than there *are* places of figures in the divisor.

2. If, after any figure has been placed in the quotient, and the operation proceeded with as directed in the foregoing rule, it should be found that the remainder is *greater* than the divisor, the quotient figure must be *increased;* and if the product of the quotient figure and the divisor is *greater* than the number from which it is to be subtracted the quotient figure must be *decreased.*

### Examples.

1. Divide 476438207 by 37.

```
                 Dividend
Divisor  37) 476438207 (12876708   Quotient.
             37
             ---
             106              12876708  Quotient
              74                    37  Divisor
             ----             --------
              324             90136956
              296            38630124
              ---            ---------
               283           476438196  Product
               259                  11  Remainder
               ---           ---------
                248          476438207  Dividend.
                222          =========
                ---
                 262
                 259
                 ---
                  307
                  296
                  ---
                   11
                  ===
```

2. Divide 73261049837 by 43.  
3. Divide 817034817520 by 78.  
4. Divide 6132715269358 by 179.  
5. Divide 746302754963 by 3578.  
6. Divide 371472693102765 by 17396.  
7. Divide 18273004936752 by 57905.  
8. Divide 263907521826147 by 274567.  
9. Divide 1932518269357251 by 814269.  
10. Divide 95049637528590 by 376047.  
11. Divide 167346003279315 by 936185.  
12. Divide 917059030526 by 176498.

When the divisor is the product of any *two* numbers *not* exceeding 12, the operation may be performed as follows:—

Divide the given quantity by one of those numbers, and the quotient resulting from such division by the other, the result will be the quotient required. If there is a remainder to each or either of the quotients, multiply the *second remainder* by the *first divisor* and add the *first remainder* to their product, the *sum* will be the *true remainder.*

### EXAMPLES.

Divide 3716955 by 56.

$$56\left\{\begin{array}{l} 7\,)\,3716955 \\ \times \\ 8\,)\ \ 530993 \quad 4 \end{array}\right.$$

$$\underline{\underline{66374}} \quad 1\times 7+4=11 \text{ Remainder}$$

13. Divide 61735047 by 42.
14. Divide 30641503 by 63.
15. Divide 87192709 by 81.
16. Divide 627013082 by 108.
17. Divide 134069135 by 132.
18. Divide 467318043 by 144.

## COMPOUND ADDITION.

COMPOUND ADDITION teaches us to collect compound numbers of the same kind into one sum.

A compound number is a number composed of several denominations, as £3 14*s.* 7¼*d.*; 3*lbs.* 11*oz.* 13*dwts.*, &c.

RULE. Place the numbers so that those of the same denomination may stand directly under each other. Add the lowest denominations as in Simple Addition, and divide their sum by as many of the same denomination as make one of the next higher; set down the remainder under that lowest denomination, and carry the quotient to the next higher denomination, which add as before, and so on to the highest, which add as in Simple Addition.

*PROOF.* The same as for Simple Addition.

# TABLES OF MONEY, WEIGHTS AND MEASURES.

---

## MONEY.

| | | | | | *written* |
|---|---|---|---|---|---|
| 2 | farthings | make | 1 | halfpenny, | ½*d.* |
| 4 | farthings | .... | 1 | penny | 1*d.* |
| 12 | pence | ...... | 1 | shilling | 1*s.* |
| 20 | shillings | .... | 1 | pound | 1£. |

All the monetary transactions of the country are carried on by means of the coins in the preceding table. There are however, in common use, several other divisions of the pound *sterling*, (called sterling to distinguish it from *nominal* money, as, Stock, Bills, Notes, &c.)

*viz:*—

| | £ | *s.* | *d.* |
|---|---|---|---|
| The groat | 0 | 0 | 4 |
| The sixpence | 0 | 0 | 6 |
| The half-crown | 0 | 2 | 6 |
| The crown | 0 | 5 | 0 |
| The half-sovereign | 0 | 10 | 0 |

There are also other coins, occasionally met with, but not considered as the *current* money of the realm,

*viz:*—

| | £ | *s.* | *d.* |
|---|---|---|---|
| The seven shilling piece | 0 | 7 | 0 |
| The half-guinea | 0 | 10 | 6 |
| The guinea | 1 | 1 | 0 |

The pound *sterling* is represented by the gold coin called a *sovereign*, which weighs 5 dwt. 3·274 grains of *standard* gold. Standard gold contains $\frac{11}{12}$ths of *pure* gold, and $\frac{1}{12}$th of alloy. The alloy used is copper. One pound weight of standard silver contains 11 oz. 2 dwts. of *pure* silver, and 18 dwts. of copper or alloy.

## TROY WEIGHT.

| | | *written* |
|---|---|---|
| 24 grains make .. | 1 penny-weight, | 1 *dwt.* |
| 20 penny-weights | 1 ounce ...... | 1 *oz.* |
| 12 ounces ...... | 1 pound ...... | 1 *lb.* |

By this weight are weighed gold, silver, and most liquids which are sold by weight.

"The Troy Weight appeared to us to be the ancient weight of this kingdom, having existed in the same state from the time of Edward the Confessor; and there are reasons to believe that the word *Troy*, has no reference to any town in France, but rather to the monkish name given to London of Troy-novant, founded on the legend of Brute: Troy Weight, therefore, according to this etymology, is, in fact, London Weight." *Report of the Commissioners of Weights and Measures.*

The legend referred to asserts that Brute, a lineal descendant of Æneas, came to England about the year 2855 B. C., where he founded the City of London, then called Trinovantum, which name was afterwards corrupted into Troy-novant.

Jewellers weigh diamonds and other precious stones by carats and grains.

| | |
|---|---|
| 4 diamond grains make.. | 1 carat |
| 151½ carats .............. | 1 ounce Troy, |

hence the diamond carat is 3 ⅙ grains nearly.

The term carat, when used to express the fineness of gold, has a relative meaning only. The whole mass of alloyed gold is supposed to be divided into 24 equal parts, thus the standard for gold coin is 22 carats fine, that is, it consists of 22 parts of pure gold, and 2 parts of alloy.

## AVOIRDUPOIS WEIGHT.

| | | | *written* |
|---|---|---|---|
| 16 | drachms make | 1 ounce | 1 *oz.* |
| 16 | ounces | 1 pound | 1 *lb.* |
| 28 | pounds | 1 quarter | 1 *qr.* |
| 4 | quarters, or 112 lbs. | 1 hundred-weight | 1 *cwt.* |
| 20 | hundred-weight | 1 ton | 1 *ton.* |

"We found the Avoirdupois Weight by which all heavy goods have been for a long time weighed (probably derived from *avoirs* the ancient name for goods or chattels, and *poids* weight) to be universally used through the kingdom." *Report of the Commissioners of Weights and Measures.*

The pound Avoirdupois is, by Act of Parliament, ordered to contain 7000 grains Troy.

By this weight are weighed all metals, gold and silver excepted, grocery, butchers'-meat, bread, corn, drugs, &c.

## WOOL WEIGHT.

| | | |
|---|---|---|
| 7 | lbs. make | 1 clove |
| 2 | cloves or 14 lbs. | 1 stone |
| 2 | stones or 28 lbs. | 1 tod |
| 6½ | tods | 1 wey |
| 2 | weys | 1 sack |
| 12 | sacks | 1 last. |

N.B. In some places, 7 tods make 1 wey, and 240 pounds 1 pack.

## APOTHECARIES' WEIGHT.

| | | | *written* |
|---|---|---|---|
| 20 | grains make | 1 scruple | 1 *scr.* |
| 3 | scruples | 1 drachm | 1 *dr.* |
| 8 | drachms | 1 ounce | 1 *oz.* |
| 12 | ounces | 1 pound | 1 *lb.* |

This weight is used exclusively in preparing medical prescriptions, and is merely a peculiar subdivision of the pound Troy; the pound, ounce, and grain of the Apothecaries being exactly the same as the pound, ounce, and grain Troy.

## CLOTH MEASURE.

| | | | written |
|---|---|---|---|
| 2¼ | inches make | 1 nail | 1 *n.* |
| 4 | nails | 1 quarter | 1 *qr.* |
| 4 | quarters | 1 yard or 36 inches | 1 *yd.* |
| 3 | quarters | 1 ell Flemish | 1 *e.Fl.* |
| 5 | quarters | 1 ell English | 1 *e.E.* |
| 6 | quarters | 1 ell French | 1 *e.Fr.* |

The yard and its subdivisions are used in measuring all sorts of wrought silks, cottons, linens, &c. The ell as a measure of length appears to be now out of use. Merchants in London, who deal in French silks, merinoes, &c., sell their goods by a measure of 44 inches called an ell.

## LONG MEASURE.

| | | | written |
|---|---|---|---|
| 3 | barley-corns make | 1 inch | 1 *in.* |
| 12 | inches | 1 foot | 1 *ft.* |
| 3 | feet | 1 yard | 1 *yd.* |
| 5½ | yards | 1 rod, pole, or perch | 1 *po.* |
| 40 | poles | 1 furlong | 1 *fur.* |
| 8 | furlongs or 1760 yds | 1 mile | 1 *m.* |
| 3 | miles | 1 league | 1 *lea.* |
| 69½ | English statute miles | 1 degree | 1° |

Long measure is used to measure the distance of places, or any thing else where length is considered without regard to breadth.

The following measure is used for the purpose of Land Surveying.

| | | |
|---|---|---|
| 100 | links make | 1 chain of 22 yards |
| 10 | chains | 1 furlong |
| 80 | chains | 1 mile. |

Sailors also use the following for nautical purposes.

| | | | |
|---|---|---|---|
| 6 | feet make .......... | 1 | fathom |
| 1012½ | fathoms or 2025 yards | 1 | nautical mile |
| 3 | nautical miles ...... | 1 | league |
| 60 | nautical miles ...... | 1 | degree. |

## SQUARE MEASURE.

| | | | | written |
|---|---|---|---|---|
| 144 | square inches make | 1 | square foot | 1 *sq. ft* |
| 9 | square feet ...... | 1 | square yard | 1 *sq. yd.* |
| 30¼ | square yards .... | 1 | square pole | 1 *sq. pole* |
| 272¼ | square feet ...... | 1 | square rod | 1 *sq. rod* |

Square measure is used to measure all kinds of surfaces or areas, such as land, flooring, plastering, slating, tiling, &c., &c. An *area* has length and breadth only.

## LAND MEASURE.

| | | | | written |
|---|---|---|---|---|
| 40 | square poles make | 1 | rood .... | 1 *ro.* |
| 4 | roods .......... | 1 | acre .... | 1 *a.* |
| 160 | square poles .... | 1 | acre | |
| 4840 | square yards .... | 1 | acre | |

## CUBIC, OR SOLID MEASURE.

| | | | | written |
|---|---|---|---|---|
| 1728 | cubic inches make | 1 | cubic foot | 1 *cu. ft.* |
| 27 | cubic feet ...... | 1 | cubic yard | 1 *cu. yd.* |

Cubic measure is used to measure the contents of all kinds of solids. A *solid* contains length, breadth, and thickness or depth.

## WINE MEASURE.

| | | | | written |
|---|---|---|---|---|
| 4 | gills make.......... | 1 | pint.......... | 1 *pt.* |
| 2 | pints .............. | 1 | quart ........ | 1 *qt.* |
| 4 | quarts ........... | 1 | gallon........ | 1 *gal.* |

### WINE MEASURE CONTINUED.

| | | *written* |
|---|---|---|
| 10 gallons | 1 anker of brandy | 1 *ank.* |
| 42 gallons | 1 tierce | 1 *tier.* |
| 63 gallons | 1 hogshead | 1 *hhd.* |
| 84 gallons | 1 puncheon | 1 *pun.* |
| 2 hogsheads or 126 gal. | 1 pipe or butt | 1 *pipe* |
| 2 pipes or 252 galls. | 1 tun | 1 *tun.* |

By this measure, wine, spirits, perry, cyder, mead, vinegar, whale and olive oil, &c., are measured.

The *Imperial gallon,* as ordered by Act of Parliament, contains 277·274 cubic inches or 277¼ cubic inches nearly, and is used for both Wine, &c. and Beer.

## BEER MEASURE.

| | | *written* |
|---|---|---|
| 2 pints make | 1 quart | 1 *qt.* |
| 4 quarts | 1 gallon | 1 *gal.* |
| 8 gallons | 1 firkin of ale | 1 *fir. ale* |
| 9 gallons | 1 firkin of beer | 1 *fir. br.* |
| 2 firkins or 18 galls. | 1 kilderkin | 1 *kil.* |
| 2 kilderkins or 36 galls. | 1 barrel | 1 *bar.* |
| 3 kilderkins or 54 galls. | 1 hogshead | 1 *hhd.* |
| 3 barrels or 108 galls. | 1 butt | 1 *bt.* |

## DRY MEASURE.

| | | *written* |
|---|---|---|
| 2 pints make | 1 quart | 1 *qt.* |
| 2 quarts | 1 pottle | 1 *pot.* |
| 2 pottles | 1 gallon | 1 *gal.* |
| 2 gallons | 1 peck | 1 *pk.* |
| 4 pecks | 1 bushel | 1 *bush.* |
| 8 bushels | 1 quarter | 1 *qr.* |
| 4 bushels | 1 coomb | 1 *cb.* |
| 5 quarters or 40 bush. | 1 wey or load | 1 *ld.* |
| 2 weys or 10 qrs. | 1 last | 1 *lst.* |
| 36 bushels | 1 chaldron | 1 *chal.* |

By this measure, coals, salt, lead-ore, oysters, corn, seeds, fruits, roots, and other dry goods, are measured.

The *Imperial bushel* contains 8 imperial gallons, or 2218·192 cubic inches, and is used for measuring corn, malt, &c.

The *Imperial heaped bushel* as used for coals, contains 2815·4887 cubic inches. Coals are now generally sold by weight, 1¼ tons being considered equal to a chaldron.

## TIME.

| | | | *written* |
|---|---|---|---|
| 60 | seconds make | 1 minute | 1 *m.* |
| 60 | minutes | 1 hour | 1 *h.* |
| 24 | hours | 1 day | 1 *d.* |
| 7 | days | 1 week | 1 *w.* |
| 4 | weeks | 1 month | 1 *m.* |
| 52 | weeks and 1 day | 1 year | 1 *yr.* |
| 12 | calendar months | 1 year | 1 *yr.* |
| 13 | lunar months and 1 day | 1 year | 1 *yr.* |
| 365 | days 5 *hrs.* 48 *m.* 55 *sec.* | 1 solar year | 1 *s. yr.* |

A *lunar month* consists of 28 days nearly, being the time in which the moon completes one revolution round the earth.

A *solar year* is the period in which the earth completes one revolution round the sun.

The length of the calendar months are *unequal,* and are usually remembered by means of the following *doggrel* lines.

Thirty days have September,
April, June, and November;
February hath twenty-eight alone,
And all the rest have thirty-one.
Except in Leap-year, and then 's the time,
When February's days are twenty-nine.

*Leap-year* is every fourth year, when 366 days are made to constitute a year, for 365¼ days nearly, being the length of a solar year. If 365 days only were counted, we should lose 6 hours nearly in every year, or 1 day nearly in every four years; the month of February is therefore, every fourth year, made to consist of 29 days instead of 28 days, except when the numbers denoting the centuries, as 16, 17, 18, 19, 20, denoting 1600, 1700, 1800, 1900, 2000, are not divisible by 4. Hence 1600 and 2000 would be leap-years, 16 and 20 being divisible by 4; and 1700, 1800, 1900 would not be leap-years, 17, 18, and 19 not being divisible by 4. This omission of the leap-year at the years 1700, 1800, &c. was first introduced into Europe in the year 1582 by Pope Gregory, (the calendar was from him called the *Gregorian Calendar*), and was made to remedy the error that would arise from considering the year as 365¼ days exactly, instead of 365 days, 48 minutes and a fraction, which error would be about *one day* in every 130 years, or 3 days and a fraction in every 400 years.

This correction of the calendar was not made in England until the year 1752, when the error was found to amount to 11 days; and this corrected calendar is called the *New Style*, to distinguish it from the *Old Style*, or *Julian Calendar*.

The Julian Calendar made the year to consist of 365¼ days *exactly*, and was called *Julian*, from its having existed from the time of Julius Cæsar; the Julian Calendar made *every* fourth year a leap-year.

The leap-year is known by the number expressing the year being exactly divisible by 4; thus the numbers 1840, 1844, 1848 being exactly divisible by 4, the years 1840, 1844, and 1848 are leap-years.

The New Style is now 12 days before the Old Style, since the year 1800 would, by the Old Style, have been a leap-year, which by the New Style, it was not; this number of 12 days constitutes the difference between Old Michaelmas Day, Old Christmas Day, &c. and our Michaelmas day, &c.

# A TABLE OF MISCELLANEOUS ARTICLES.

| | |
|---|---|
| A firkin of butter is | 56 lbs. |
| A firkin of soap | 64 lbs. |
| A barrel of pot-ashes | 200 lbs. |
| A barrel of anchovies | 30 lbs. |
| A barrel of soap | 256 lbs. |
| A barrel of raisins | 112 lbs. |
| A fother of lead in London, 20 cwt., or | 2240 lbs. |
| A stone of iron | 14 lbs. |
| A stone of butcher's meat | 8 lbs. |
| A stone of glass | 5 lbs. |
| A stone of hemp | 32 lbs. |
| A stone of cheese | 16 lbs. |
| A gallon of train and other oils | 7½ lbs. |
| A faggot of steel | 120 lbs. |
| A seam of glass is 24 stone of 5 lbs., or | 120 lbs. |
| A peck of salt | 14 lbs. |
| A bushel of salt | 56 lbs. |
| A quintal | 100 lbs. |
| A chest of tea, about | 84 lbs. |
| A bag of rice | 168 lbs. |
| A hogshead of pilchards (about 3000 fish) | 40 gals. |
| A tun of seed oil | 236 gals. |
| A tun of fish oil | 252 gals. |
| A ton of salt | 42 bush. |

| | | |
|---|---|---|
| 56 lbs. of old hay<br>60 lbs. of new hay | make | 1 truss |
| 36 lbs. of straw | | 1 truss |
| 36 trusses of hay or straw | | 1 load |
| 12 articles | | 1 dozen |
| 12 dozen | | 1 gross |
| 20 articles | | 1 score |
| 5 score | | 1 common hundred |
| 6 score | | 1 great hundred |
| 30 deals | | 1 quarter |
| 4 quarters | | 1 hundred |
| 24 sheets of paper | | 1 quire |

| | |
|---|---|
| 25 sheets ................. | 1 quire (Printer's) |
| 20 quires ................. | 1 ream |
| 21½ quires ................. | 1 Printer's ream. |

Lemons, oranges, corks, and some other articles, are often sold by the gross; nails, tacks, &c., have six score to the hundred.

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## EXAMPLES.

### MONEY.

| | (1) £ | s. | d. | (2) £ | s. | d. | (3) £ | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| | 37 | 12 | 4¼ | 89 | 19 | 11¾ | 137 | 19 | 7½ |
| | 49 | 17 | 8½ | 76 | 7 | 8 | 84 | 18 | 8 |
| | 39 | 9 | 11 | 43 | 16 | 2½ | 369 | 17 | 9¾ |
| | 68 | 13 | 2¾ | 27 | 13 | 7 | 25 | 16 | 10½ |
| | 47 | 0 | 0 | 39 | 8 | 11¾ | 37 | 15 | 11¾ |
| | 35 | 18 | 9¾ | 18 | 3 | 9¼ | 832 | 14 | 1½ |
| | 52 | 3 | 10½ | 52 | 15 | 6 | 76 | 13 | 2¼ |
| Sum | 330 | 15 | 10¾ | | | | | | |
| | 293 | 3 | 6½ | | | | | | |
| Proof | 330 | 15 | 10¾ | | | | | | |

| (4) £ | s. | d. | (5) £ | s. | d. | (6) £ | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 147 | 13 | 4¾ | 369 | 18 | 7½ | 315 | 17 | 5¼ |
| 836 | 11 | 1¼ | 138 | 17 | 1¼ | 196 | 11 | 11¾ |
| 275 | 14 | 11 | 75 | 13 | 0 | 235 | 7 | 8 |
| 96 | 12 | 7½ | 427 | 8 | 9¾ | 67 | 10 | 0¼ |
| 184 | 7 | 9 | 56 | 16 | 2½ | 8 | 8 | 0½ |
| 7 | 10 | 2¾ | 139 | 4 | 10¾ | 414 | 14 | 11 |
| 15 | 6 | 0¼ | 48 | 9 | 7 | 79 | 19 | 7¾ |

## TROY WEIGHT.

| (7) *lb.* | *oz.* | *dwt.* |
|---|---|---|
| 375 | 11 | 19 |
| 49 | 9 | 18 |
| 618 | 11 | 13 |
| 27 | 7 | 7 |
| 135 | 0 | 14 |
| 48 | 4 | 13 |
| 362 | 3 | 8 |

| (8) *oz.* | *dwt.* | *gr.* |
|---|---|---|
| 23 | 19 | 23 |
| 49 | 13 | 14 |
| 7 | 17 | 21 |
| 16 | 14 | 17 |
| 4 | 11 | 8 |
| 13 | 17 | 21 |
| 89 | 15 | 9 |

| (9) *lb.* | *oz.* | *dwt.* |
|---|---|---|
| 714 | 3 | 7 |
| 83 | 11 | 19 |
| 269 | 4 | 6 |
| 814 | 2 | 15 |
| 943 | 7 | 14 |
| 65 | 0 | 3 |
| 8 | 8 | 8 |

## AVOIRDUPOIS WEIGHT.

| (10) *tons.* | *cwt.* | *qr.* | *lb.* |
|---|---|---|---|
| 17 | 19 | 3 | 27 |
| 13 | 3 | 2 | 18 |
| 4 | 18 | 1 | 6 |
| 89 | 7 | 2 | 24 |
| 65 | 15 | 3 | 19 |
| 47 | 4 | 1 | 5 |
| 3 | 9 | 0 | 18 |

| (11) *cwt.* | *qr.* | *lb.* | *oz.* | *dr.* |
|---|---|---|---|---|
| 314 | 3 | 27 | 15 | 15 |
| 751 | 1 | 18 | 11 | 11 |
| 835 | 2 | 26 | 13 | 7 |
| 26 | 2 | 9 | 11 | 5 |
| 148 | 0 | 0 | 8 | 14 |
| 27 | 0 | 4 | 3 | 5 |
| 185 | 3 | 18 | 9 | 12 |

## APOTHECARIES' WEIGHT.

| (12) *lb.* | *oz.* | *dr.* | *sc.* |
|---|---|---|---|
| 13 | 11 | 7 | 2 |
| 185 | 7 | 6 | 0 |
| 26 | 0 | 5 | 1 |
| 15 | 4 | 4 | 0 |
| 198 | 11 | 3 | 2 |
| 27 | 3 | 2 | 2 |
| 185 | 7 | 1 | 1 |

| (13) *oz.* | *dr.* | *sc.* | *gr.* |
|---|---|---|---|
| 735 | 7 | 2 | 19 |
| 81 | 1 | 1 | 13 |
| 7 | 6 | 0 | 15 |
| 49 | 0 | 0 | 7 |
| 635 | 4 | 2 | 9 |
| 19 | 3 | 0 | 11 |
| 48 | 5 | 2 | 14 |

## CLOTH MEASURE.

| (14) *yds.* | *qr.* | *n.* | (15) *E.e.* | *qr.* | *n.* | (16) *Fr.e.* | *qr.* | *n.* |
|---|---|---|---|---|---|---|---|---|
| 14 | 3 | 3 | 48 | 3 | 3 | 173 | 5 | 3 |
| 13 | 0 | 2 | 27 | 4 | 1 | 69 | 2 | 2 |
| 17 | 1 | 1 | 13 | 1 | 2 | 48 | 4 | 1 |
| 15 | 3 | 0 | 54 | 0 | 1 | 63 | 1 | 2 |
| 8 | 2 | 2 | 69 | 3 | 0 | 15 | 0 | 0 |
| 16 | 0 | 3 | 48 | 2 | 2 | 8 | 4 | 1 |
| 5 | 1 | 2 | 17 | 4 | 2 | 17 | 3 | 2 |

## LONG MEASURE.

| (17) *mi.* | *fur.* | *po.* | (18) *yds.* | *ft.* | *in.* | (19) *ft.* | *in.* | *b.c.* |
|---|---|---|---|---|---|---|---|---|
| 184 | 7 | 19 | 17 | 2 | 11 | 36 | 11 | 2 |
| 35 | 6 | 17 | 14 | 1 | 7 | 17 | 4 | 1 |
| 26 | 0 | 19 | 9 | 0 | 9 | 19 | 10 | 1 |
| 143 | 4 | 13 | 7 | 1 | 10 | 48 | 3 | 0 |
| 25 | 2 | 39 | 135 | 0 | 0 | 3 | 5 | 2 |
| 19 | 3 | 27 | 26 | 2 | 4 | 1 | 11 | 1 |
| 248 | 5 | 15 | 18 | 1 | 5 | 19 | 7 | 2 |

## LAND MEASURE.

| (20) *a.* | *r.* | *p.* | (21) *a.* | *r.* | *p.* |
|---|---|---|---|---|---|
| 19 | 1 | 19 | 37 | 2 | 7 |
| 35 | 3 | 15 | 14 | 3 | 19 |
| 27 | 2 | 37 | 15 | 1 | 7 |
| 19 | 1 | 18 | 18 | 0 | 32 |
| 45 | 0 | 32 | 36 | 0 | 24 |
| 76 | 3 | 15 | 47 | 2 | 7 |
| 19 | 1 | 16 | 19 | 1 | 19 |

## WINE MEASURE.

| (22) *hhd.* | *gal.* | *qts.* |
|---|---|---|
| 43 | 47 | 3 |
| 28 | 36 | 1 |
| 16 | 54 | 1 |
| 35 | 38 | 0 |
| 27 | 41 | 2 |
| 18 | 7 | 3 |
| 5 | 16 | 1 |

| (23) *tun.* | *hhd.* | *gal.* | *qts.* |
|---|---|---|---|
| 183 | 1 | 37 | 1 |
| 26 | 3 | 19 | 3 |
| 13 | 2 | 48 | 2 |
| 159 | 1 | 27 | 0 |
| 26 | 0 | 15 | 2 |
| 148 | 3 | 43 | 3 |
| 35 | 2 | 27 | 2 |

## DRY MEASURE.

| (24) *qrs.* | *b.* | *p.* |
|---|---|---|
| 314 | 7 | 3 |
| 27 | 2 | 1 |
| 13 | 6 | 0 |
| 487 | 3 | 3 |
| 35 | 1 | 2 |
| 246 | 7 | 3 |
| 18 | 5 | 1 |

| (25) *load* | *qr.* | *b.* |
|---|---|---|
| 17 | 4 | 7 |
| 13 | 1 | 1 |
| 18 | 0 | 6 |
| 49 | 3 | 3 |
| 7 | 2 | 5 |
| 16 | 3 | 2 |
| 5 | 2 | 4 |

## TIME.

| (26) *yrs.* | *m.* | *d.* |
|---|---|---|
| 143 | 12 | 23 |
| 26 | 11 | 4 |
| 19 | 4 | 19 |
| 15 | 10 | 16 |
| 183 | 7 | 13 |
| 25 | 11 | 24 |
| 69 | 12 | 17 |

| (27) *m.* | *d.* | *h.* |
|---|---|---|
| 11 | 17 | 23 |
| 14 | 14 | 17 |
| 37 | 23 | 14 |
| 18 | 18 | 8 |
| 26 | 25 | 6 |
| 14 | 7 | 13 |
| 13 | 9 | 8 |

| (28) *d.* | *h.* | *m.* |
|---|---|---|
| 138 | 17 | 39 |
| 26 | 9 | 51 |
| 15 | 11 | 56 |
| 49 | 12 | 9 |
| 138 | 8 | 57 |
| 47 | 14 | 18 |
| 19 | 7 | 19 |

## COMPOUND SUBTRACTION.

**Compound Subtraction** teaches us to find the difference of any two compound numbers of the same kind.

**Rule.** Place the lesser number under the greater, as in Addition, then, beginning with the *lowest* denomination, subtract the lower number from the upper, and set down the remainder; but if the lower number be *greater* than the upper, borrow as many of the *same* denomination as make *one* of the next higher, and add it to the upper; subtract the lower number from this sum, and set down the remainder, observing, in this case, to carry *one* to the number to be subtracted in the next higher denomination.

**Proof.** The same as for Simple Subtraction.

### EXAMPLES.

#### MONEY.

|  | (1) £ | s. | d. | (2) £ | s. | d. | (3) £ | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
|  | 37 | 19 | 4½ | 683 | 13 | 2¼ | 326 | 7 | 6¼ |
|  | 19 | 17 | 5¼ | 527 | 14 | 7½ | 273 | 15 | 4¾ |
| Rem. | 18 | 1 | 11¼ |  |  |  |  |  |  |
| Proof | 37 | 19 | 4½ |  |  |  |  |  |  |

| (4) £ | s. | d. | (5) £ | s. | d. | (6) £ | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 738 | 17 | 4¼ | 361 | 2 | 4¼ | 176 | 17 | 3 |
| 75 | 19 | 8 | 173 | 13 | 8¾ | 35 | 4 | 11¼ |

#### TROY WEIGHT.

| (7) lb. | oz. | dwt. | (8) oz. | dwt. | gr. | (9) lb. | oz. | dwt. |
|---|---|---|---|---|---|---|---|---|
| 183 | 7 | 13 | 37 | 17 | 12 | 48 | 2 | 11 |
| 49 | 11 | 17 | 18 | 13 | 19 | 9 | 7 | 13 |

## AVOIRDUPOIS WEIGHT.

| (10) | *cwt.* | *qr.* | *lb.* | (11) | *qr.* | *lb.* | *oz.* | (12) | *lb.* | *oz.* | *dr.* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 175 | 2 | 4 | | 87 | 21 | 1 | | 95 | 2 | 11 |
| | 35 | 3 | 19 | | 14 | 27 | 11 | | 38 | 7 | 14 |

## APOTHECARIES' WEIGHT.

| (13) | *lb.* | *oz.* | *dr.* | (14) | *oz.* | *dr.* | *sc.* | (15) | *dr.* | *sc.* | *gr.* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 47 | 3 | 2 | | 14 | 3 | 2 | | 135 | 1 | 13 |
| | 14 | 7 | 7 | | 7 | 4 | 2 | | 28 | 2 | 15 |

## CLOTH MEASURE.

| (16) | *yds.* | *qr.* | *n.* | (17) | *E.e.* | *qr.* | *n.* | (18) | *Fr.e.* | *qr.* | *n.* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 93 | 2 | 1 | | 187 | 4 | 2 | | 835 | 1 | 1 |
| | 45 | 3 | 3 | | 145 | 1 | 1 | | 496 | 5 | 3 |

## LONG MEASURE.

| (19) | *mi.* | *fur.* | *po.* | (20) | *yds.* | *ft.* | *in.* | (21) | *ft.* | *in.* | *b.c.* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 83 | 3 | 4 | | 138 | 1 | 7 | | 347 | 3 | 1 |
| | 19 | 7 | 23 | | 29 | 2 | 11 | | 19 | 5 | 2 |

## LAND MEASURE.

| (22) | *a.* | *r.* | *p.* | (23) | *a.* | *r.* | *p.* | (24) | *a.* | *r.* | *p.* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 183 | 1 | 19 | | 45 | 2 | 15 | | 352 | 1 | 17 |
| | 175 | 3 | 18 | | 41 | 3 | 27 | | 148 | 1 | 19 |

## WINE MEASURE.

| (25) | *hhd.* | *gal.* | *qt.* | (26) | *pipe* | *hhd.* | *gal.* | (27) | *tun* | *hhd.* | *gal* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 54 | 17 | 1 | | 64 | 1 | 47 | | 35 | 1 | 47 |
| | 27 | 23 | 2 | | 38 | 0 | 52 | | 19 | 3 | 54 |

### DRY MEASURE.

| (28) qrs. | b. | p. | (29) gal. | qt. | pt. | (30) qrs. | b. | p. |
|---|---|---|---|---|---|---|---|---|
| 69 | 3 | 1 | 315 | 2 | 1 | 93 | 1 | 1 |
| 15 | 6 | 2 | 49 | 3 | 1 | 49 | 5 | 2 |

### TIME.

| (31) m. | d. | h. | (32) yrs. | m. | d. | (33) hrs. | m. | sec. |
|---|---|---|---|---|---|---|---|---|
| 135 | 21 | 3 | 135 | 1 | 14 | 327 | 13 | 17 |
| 121 | 18 | 17 | 69 | 11 | 17 | 19 | 51 | 54 |

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## COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION teaches us to find the sum of any compound number, repeated any given number of times.

Compound Multiplication may be divided into *three* cases, viz.—

1. When the Multiplier does *not* exceed 12.

2. When the Multiplier *is* the product of any two numbers not exceeding 12.

3. When the Multiplier *is not* the product of any two numbers not exceeding 12.

CASE 1. RULE. Place the multiplier under the *lowest* denomination of the multiplicand. Multiply the lowest denomination by the multiplier, and divide the product by as many of the *same* denomination as make *one* of the higher, set down the remainder, and add the quotient to the product of the next higher denomination found as before, and so on to the highest, the result will be the product required.

CASE 2. RULE. Multiply the given price or quantity by one of those numbers, and *that product* by the other, the result will be the product required.

CASE 3. RULE. Multiply the given price or quantity by 10, and *that product* again by 10, and so on, according to the number of places of figures contained in the multiplicand, then, (if the number is under 1000,) multiply the *hundred* times by the number of units in the *hundreds' place* of the multiplier, the *ten* times by the number of units in the *tens' place* and the *given price* or quantity by the number of *units* in the multiplier, add these several products together, and their sum will be the whole product required.

## EXAMPLES.

1. What is the price of 5 sheep at £1 17*s.* 8½*d.* per sheep?

| £ | *s.* | *d.* | |
|---|---|---|---|
| 1 | 17 | 8½ | Multiplicand |
| | | 5 | Multiplier |
| £9 | 8 | 6½ | Product. |

2. 42 yards of satin at 14*s.* 9*d.* per yard.

| | *s.* | *d.* | |
|---|---|---|---|
| | 14 | 9 | |
| | | 6×7=42 | |
| 4 | 8 | 6 | Price of 6 yards |
| | | 7 | |
| £30 | 19 | 6 | Price of 42 yards. |

3. 735 pairs of silk stockings at 7*s.* 10¼*d.*per pair.

| | *s.* | *d.* | | |
|---|---|---|---|---|
| | 7 | 10¼ | Price of | 1 pair |
| | | 10 | | |
| 3 | 18 | 6½ | Price of | 10 pairs |
| | | 10 | | |
| 39 | 5 | 5 | Price of | 100 pairs |
| | | 7 | | |
| 274 | 17 | 11 | Price of | 700 pairs |
| 11 | 15 | 7½ | Price of | 30 pairs |
| 1 | 19 | 3¼ | Price of | 5 pairs |
| £288 | 12 | 9¾ | Price of | 735 pairs. |

4. What is the price of 9 yards of silk at 4*s.* 11½*d.* per yard?

5. What is the price of 11 ounces of silver at 7*s.* 5¾*d.* per ounce?

6. What is the price of 12 gallons of brandy at £1 14*s.* 7*d.* per gallon?

7. Multiply 3 cwt. 2 qrs. 17 lb. 13 oz. 14 dr. by 7.

8. Multiply 17 yds. 3 qrs. 2 n. by 12.

9. What is the price of 36 oxen at £27 13*s.* 4½*d.* per ox?

10. What is the price of 42 ells of merino at 11*s.* 8¾*d.* per ell?

11. What is the price of 56 quarters of wheat at £3 15*s.* 10½*d.* per quarter?

12. What is the price of 72 cwt. of cheese at £5 18*s.* 3¼*d.* per cwt.?

13. What is the price of 108 dozen of port at £2 13*s*. 7*d*. per dozen ?

14. What is the price of 132 pairs of stockings at 5*s*. 9¾*d*. per pair ?

15. What is the price of 144 shawls at £1 17*s*. 11½*d*. per shawl ?

16. What is the price of 39 tons of iron at £3 13*s*. 8¼*d*. per ton ?

17. What is the price of 87 oxen at £27 18*s*. 6*d*. per ox ?

18. What is the price of 138 gallons of brandy at £1 13*s*. 4*d*. per gallon ?

19. What is the price of 117 pieces of cloth at £35 7*s*. 8½*d*. per piece ?

20. What is the price of 387 dozen of sherry at £1 17*s*. 9½*d*. per dozen ?

21. What is the price of 419 yards of satin at 16*s*. 7¾*d*. per yard ?

22. What is the price of 754 cwt. of sugar at £3 15*s*. 3¼*d*. per cwt. ?

23. What is the price of 359 dozen of candles at 6*s*. 5½*d*. per dozen ?

24. What is the price of 198 pieces of riband at £1 18*s*. 5¾*d*. per piece ?

25. What is the price of 296 sheep at £1 19*s*. 11¾*d*. per sheep ?

26. Multiply 7 cwt. 2 qrs. 13 lbs. 11 oz. by 138.

27. What is the weight of 37 hogsheads of tobacco, each weighing 7 cwt. 3 qrs. 19 lbs. ?

## COMPOUND DIVISION.

**Compound Division** teaches us to divide any compound number into any proposed number of equal parts.

**Rule.** Place the divisor to the left hand of the dividend, divide the highest denomination by the divisor, as in Simple Division, and multiply the remainder, if any, by as many of the next lower denomination as make *one* of the denomination previously divided, adding to the product the parts of the less denomination contained in the dividend, divide that sum by the divisor as before, and so on, to the lowest denomination; the quotients placed in order will be the result required.

**Proof.** The same as for Simple Division.

### Examples.

1. Divide £437 15*s.* 9¼*d.* by 7.

| | | £ | *s.* | *d.* | |
|---|---|---|---|---|---|
| Divisor | 7) | 437 | 15 | 9¼ | Dividend |
| Quotient | | 62 | 10 | 9¾ | 4 Remainder |
| | | | | 7 | |
| | | £437 | 15 | 9¼ | Proof. |

2. Divide £398 13*s.* 5¾*d.* by 54.

| | | £ | *s.* | *d.* | |
|---|---|---|---|---|---|
| 54 | 6) | 398 | 13 | 5¾ | |
| | × | | | | |
| | 9) | 66 | 8 | 10¾ | - 5 |
| | | £7 | 7 | 7¾ | - 4×6+5=29. |

*Answer* £7 7*s.* 7¾*d.* - 29.

3. Divide £4763 15*s.* 9½*d.* by 137.

```
         £     s.   d.    £    s.   d.
137 ) 4763    15   9½ ( 34   15   5¼ - 41.
      411
      ----
       653
       548
      ----
       105
        20
      ----
137 ) 2115 ( 15s.
      137
      ----
       745
       685
      ----
        60
        12
      ----
137 )  729 ( 5d.
       685
      ----
        44
         4
      ----
137 )  178 ( ¼
       137
      ----
        41
      ====
```

4. Divide £6184 17*s.* 11½*d.* by 7.
5. Divide £13756 13*s.* 4¾*d.* by 9.
6. Divide £41683 12*s.* 3¼*d.* by 12.
7. Divide £382695 13*s.* 7¾*d.* by 35.
8. Divide £213071 18*s.* 10*d.* by 132.
9. Divide 1735 cwt. 3 qrs. 17 lbs. by 9.
10. Divide 4169 yds. 3 qrs. 1 n. by 11.
11. Divide £73615 10*s.* 4¾*d.* by 375.
12. Divide £84349 0*s.* 8¼*d.* by 179.
13. Divide 73601 cwt. 3 qrs. 8 lbs. by 827.
14. If 47 oxen cost £1749 13*s.* 9*d.* what did one ox cost?

15. Bought 273 sheep for £483 19*s.*, what did one sheep cost?

16. Bought a ship load of coals containing 349 tons for £600, what did I give for one ton?

17. If I buy a piece of cloth containing 57 ells for £137 14*s.*, what do I give for one ell?

## REDUCTION.

REDUCTION teaches us to reduce quantities of one denomination to another without altering their value.

Reduction may be divided into *two* cases.

1. Where quantities of a *higher* denomination are to be reduced to a lower.

2. Where quantities of a *lower* denomination are to be reduced to a higher.

CASE 1. RULE. Quantities of a higher denomination are reduced to a lower, by *multiplying* the higher denomination by as many of the next lower as make *one* of that higher denomination, adding to the product the parts of the lower denomination, where the quantity to be reduced is compound, and so on, until the required denomination is obtained.

CASE 2. RULE. Quantities of a lower denomination are reduced to a higher, by *dividing* the lower denomination by as many of that lower as make *one* of the next higher denomination, and so on, until the required denomination is obtained.

In reducing quantities from a higher to a lower denomination, it will be sometimes necessary to multiply by the fractions ½ and ¼. When that is the case, multiply by the whole number, to which either of these fractions is annexed, as in Simple Multiplication, and then divide the multiplicand by the number in the fraction placed below the line, and add the quotient to the product obtained by multiplying by the whole number—thus, in reducing 40 poles, which make 1 mile, into yards, the poles must be multiplied by 5½, because 5½ yards make 1 pole. The operation is as follows :—

*poles*
40
5½
200 product by 5
20 product by ½
220 product by 5½.

## EXAMPLES.

1. Reduce £37 19*s*. 7¼*d*. into farthings.

£ *s*. *d*.
37 19 7¼
20
759 shillings
12
9115 pence
4
*Answer* 36461 farthings.

EXPLANATION. In the preceding example the pounds are reduced to shillings by multiplying by 20, because 20 shillings make, or are equal to (=) one pound ;—the shillings are reduced to pence by multiplying by 12, because 12 pence make one shilling ;—the pence are reduced to farthings by multiplying by 4, because 4 farthings make one penny ;—and 36461 farthings are equal in value to £37 19*s*. 7¼*d*.

2. In 7318463 farthings how many pounds sterling?

*farthings*
4) 7318463
12) 1829615¾
2,0) 15246,7 11¾*d.*
£7623 7*s.* 11¾*d.* *Answer.*

EXPLANATION. Here the farthings are reduced to pence by dividing by 4, because 4 farthings make, or are equal to (=) one penny, the quotient will be pence and the remainder farthings;—the pence are reduced to shillings by dividing by 12, because 12 pence make one shilling, the quotient will be shillings and the remainder pence;—the shillings are reduced to pounds, by dividing by 20, because 20 shillings make one pound, the quotient will be pounds, and the remainder shillings; and £7623 7*s.* 11¾*d.* are equal in value to 7318463 farthings.

3. In £19 19*s.* 11¾*d.* how many farthings?
4. Reduce £58 17*s.* 8½*d.* into farthings.
5. Reduce £47 13*s.* 4*d.* into two-pences, pence, and farthings.
6. Reduce £83 11*s.* 9*d.* into three-pences, pence, and farthings.
7. Reduce £39 19*s.* 8*d.* into groats, pence, and farthings.
8. Reduce £84 7*s.* 6*d.* into sixpences, pence, and farthings.
9. Reduce £78 15*s.* into crowns, half-crowns, sixpences, pence, and farthings.
10. In 6154973 farthings how many pounds?
11. In 76352075 farthings how many pounds?
12. In 31567603 farthings how many pounds?
13. In 3 lb. 11 oz. 13 dwt. 22 gr. how many grains?
14. In 6327519 grains how many pounds troy?
15. In 17 silver candlesticks, each weighing 2 lb. 7 oz. 13 dwt. 14 gr. how many grains?

16. Reduce 24 lb. 11 oz. 7 dr. 2 sc. 19 gr. into grains.

17. Reduce 87615327 grains into pounds (apothecaries' weight.)

18. In 27 boxes of myrrh each 37 lb. 3 oz. 4 dr. how many grains?

19. In 8754356 ounces how many cwts?

20. In 3 tons 17 cwt. 2 qr. 23 lb. 11 oz. of iron how many drachms?

21. In 11 tons 19 cwt. 3 qr. 27 lb. 15 oz. 15 dr. how many drachms?

22. In 731842695 drachms how many tons?

23. In 13 hogsheads of sugar, each weighing 4 cwt. 2 qr. 19 lb. how many pounds?

24. Reduce 363 yds. 3 qrs. 3 n. into nails.

25. Reduce 179 E.e. 4 qrs. into yards.

26. Reduce 419 Fr.e. 2 qrs. into English ells.

27. In 14 pieces of cloth, each 37 yards how many nails?

28. In 15 bales of holland, each bale 19 pieces, each piece 48 Flemish ells, how many yards?

29. In 13 mi. 7 fur. 19 p. how many poles?

30. In 4163251 yards how many miles?

31. In 50 miles how many barleycorns?

32. In 183 a. 3 r. 17 p. how many poles?

33. In. 16 hhd. 35 gal. 3 qt. of wine how many pints?

34. In 473596 gallons of brandy how many tuns?

35. In 369 qrs. 3 b. 2 p. of wheat how many pecks?

36. In 365 d. 5 h. 48 m. 55 s. which make a solar year, or the time in which the earth makes one revolution about the sun, how many seconds?

37. In 354 d. 8 h. 48 m. 36 s. how many seconds?

38. How many seconds have elapsed since the creation of the world to Christmas 1840, supposing the creation to have been 4004 years B.C. reckoning 365¼ days to the year?

# RULE OF THREE.

**The Rule of Three** teaches us to find a number that shall have the same proportion to *one* of three given numbers that there is between the other *two*, and is so called from *three* numbers being given to find a *fourth.*

As the doctrine of ratio or proportion is rather above the comprehension of the young beginner, he must rest satisfied with the following explanations and facts.

**Ratio** is the relation which one quantity bears to another quantity of the *same* kind with respect to *magnitude.*

Thus the relation between 12 and 6, where 12 contains 6 twice, is called a *ratio* and is *written* 12 : 6, and *read* twelve to six.

"It is evident that this relation can only exist between quantities of a similar kind; thus a number must be compared with a number; a line with a line; &c. &c.: and it would be absurd to compare a certain number of feet with a certain number of pounds; &c. &c." *Bridges' Algebra.*

**Proportion** is the equality of ratios. For, four quantities are said to be proportional when the first contains, or is contained in, the second, *as often as* the third contains, or is contained in, the fourth.

Now, 12 contains 6, as often as 4 contains 2;
For 12 is twice 6, and 4 is twice 2.

And the proportion between these four numbers, 12, 6, 4, and 2, is *written* thus—12 : 6 :: 4 : 2, and *read,* as twelve is to six, so is four to two, where it is evident that the ratio of 12 : 6 is equal to the ratio of 4 : 2.

When four quantities are proportional, the product of the first term multiplied by the fourth, is *equal* to the product of the second multiplied by the third, and consequently the product of second and third divided by the first will give the fourth, from whence we derive the principle upon which the following rule is founded.

**Rule.** Place that number which is of the *same* kind with the number required in the *third* term; then if the number required will be *greater* than that third term, place the *greater* of the other two numbers in the second term and the less in the first; but if *less* than the third term, then place the *less* in the second, and the greater in the first.

Having thus stated the question, reduce the first and second terms to the lowest denomination mentioned in either of them, and the third term to the lowest denomination mentioned in it.

Multiply the second and third terms, so reduced, together, and divide their product by the first term reduced, the quotient or number required will be in the same denomination that the third term was reduced to, this quotient must be again reduced, if necessary, to the highest denomination it admits of.

**Observation.** When the first term is an unit (1), the product will be the same as the quotient, as any number divided by an unit (1) remains the same: and the product of the second and third terms must be always considered of the same denomination that the third term was reduced to.

In working an example in the Rule of Three, the beginner should always ask himself the following questions:—

1. Which of the three given numbers is of the same kind with the number required?
2. Will the number required be greater or less than the third term?
3. What is the lowest denomination mentioned in the first and second terms?
4. What is the lowest denomination mentioned in the third term?

## EXAMPLES.

1. If a yard of cloth cost 11*s.* 4*d.*, what will a piece of the same cloth cost which measures 39½ yards?

*yd.* *yds.* *qrs.* *s.* *d.*

As 1 : 39 2 :: 11 4 : Number required.

4 4 12

4 158 136

136

948

474

158

4) 21488

12) 5372

2,0) 44,7 . 8

£22 . 7*s.* . 8*d.* Number required.

*Answer* £22 7*s.* 8*d.*

EXPLANATION. In the preceding example, the number required will be in money, therefore the given number containing money, namely 11*s.* 4*d.* must be placed in the third term. The number required, being the price of 39½ yards, will be greater than 11*s.* 4*d.*, the price of 1 yard, because 39½ yards are greater than 1 yard; therefore the greater 39½ yards must be placed in the sceond term, and 1 yard in the first term. Again, quarters of yards are the lowest denomination mentioned in the first and second terms, therefore both 1 yard and 39½ yards must be reduced to quarters of yards; and pence being the lowest denomination mentioned in the third term, therefore 11*s.* 4*d.* must be reduced to pence. The second and third terms so reduced, namely 158 and 136 are to be multiplied together, and their product 21488 is to be divided by 4, the first term reduced; the quotient will be 5372 pence, in the same denomination that the third term was reduced to, which quotient, 5372 pence, is equal to £22 7*s*. 8*d*. and will be the number, or price of the 39¼ yards, required.

2. If I buy 1 cwt. of sugar for £4 13*s*. 4*d*., how much can I buy for £30 at the same rate?

```
      £   s.   d.      £        cwt.
As 4  13   4  :  30   ::  1   :  Number required.
     20              20
     ——              ———
     93             600
     12              12
   ————            ————
   1120     1120) 7200 (6cwt. 1qr. 20lb. num. req.
                  6720
                  ————
                   480
                     4
                  ————
            1120) 1920 (1qr.
                  1120
                  ————
                   800
                    28
                  ————
                  6400
                 1600
                 —————
         1120 ) 22400 (20lb.
                 2240
                 ————
                    0
                 ====
```

*Answer* 6 cwt. 1 qr. 20 lb.

EXPLANATION. In this example, the third term being an unit (1), the product of the second and third terms will be the same as the second term, namely 7200, and since the third term was in hundred-weights, 7200 cwts. The quotient is then found as in Compound Division.

3. What is the price of 1 ton 5 cwt. of coals at 1*s*. 4*d*. per cwt.?

4. If 1 cwt. of sugar cost £4 14*s*. 6*d*. what did 1 pound cost?

5. If a tod of wool cost £1 17*s*. 4*d*. what did 3 cwt. 2 qrs. 13 lbs. cost?

6. What is the value of 3 qrs. 1 n. of satin at 15*s*. 3*d*. per yard?

7. If a gentleman's income is £1260 5*s*. 5*d*. per annum, what is that per day?

8. If 32 men can mow a piece of ground in 4 days, how long would 18 men be mowing the same?

9. If 23 yds. 3 qrs. of silk will line a piece of cloth that is 17 yds. 1 qr. long, and 1½ yd. wide, what is the width of the silk?

10. How many yards of carpet that is 2 feet 3 in. wide will cover a room that is 21 feet long and 16 feet wide?

11. How many quarters of corn will £50 purchase at the rate of 7*s*. 9*d*. per bushel?

12. If a poor's rate of 2*s*. 3*d*. in the pound be granted, what must a farmer pay towards it, who is rated at £749 13*s*. 4*d*.?

13. What is the price of 4 silver candlesticks, each weighing 14 oz. 13 dwt. 19 gr. at the rate of 7*s*. 3½*d*. per ounce?

14. If 2 cwt. 3 qrs. of sugar cost £9 15*s*. 6*d*. what will 23 cwt. 1 qr. 15 lb. cost at the same rate?

15. If a servant's wages are 18 guineas per year what will he receive for 247 days service?

16. If the penny loaf weighs 12½ ounces when flour is £2 10*s*. 6*d*. per sack, what will it weigh when flour is £3 15*s*. 4*d*. per sack?

17. Bought 15 pockets of hops each weighing 2 cwt. 1 qr. 7 lb. what is their cost at £8 15*s*. 4*d*. per cwt.?

18. If 2 yds. 3 qrs. of broad cloth that is 1 yd. 3 qrs. wide will make a suit of clothes, how much cloth 1 yd. 1 qr. wide will be required for the same purpose?

19. If a person saves 3*s*. 7*d*. per week, how long will he be saving £100?

20. The whole rental of a parish is £5362, the expence of maintaining the poor is £735 13*s*. 8*d*. what

poor's rate, which is so much for every pound of the rental, must be paid to meet this expence?

21. The rental of an estate is £1738 16*s*., the land tax is 3½*d*. in the pound, what is the owner's income?

22. If an ounce of gold be worth £3 19*s*. 10½*d*., what is the worth of a wedge weighing 9 lb. 10 oz. 13 dwt. 4 gr.?

23. A tradesman owes a merchant £538 13*s*. 7*d*., but becoming a bankrupt, his assets will only pay a dividend of 4*s*. 3¼*d*. in the pound, what will the merchant receive?

24. An oilman bought 4 tuns of oil for £351 13*s*. 9*d*., but, through the carelessness of his carman, one tun was damaged, and 59 gallons were lost; at what price must the oilman sell the remainder per gallon so as neither to gain or lose by his bargain?

25. A bankrupt's debts are £793 18*s*. 4*d*. his assets are £378 13*s*., what will his creditors receive in the pound?

26. A gentleman's income is £1083 1*s*., what sum may he spend per day, so that he may lay by 300 guineas at the year's end?

27. A tradesman buys 318 yards of cloth for £111 6*s*., and retails it at 8*s*. 3*d*. per yard, what does he gain?

28. If I buy 189 yards of satin for £73 13*s*. 6*d*., at what price must I sell it per yard, so as to gain 20 guineas by my bargain?

## FRACTIONS.

A **Fraction** is any part or parts of an integer.

An *integer* is any *whole* quantity, or any quantity *considered* as a whole.

Fractions are of *two* kinds, Vulgar or common, and Decimal Fractions.

## VULGAR FRACTIONS.

A **Vulgar Fraction** is represented by *two* numbers placed the one above the other, with a line drawn between them, thus $\frac{3}{4}$, which is read three-fourths.

The number *below* the line is called the ***denominator*** and shews into how many *equal* parts the integer is divided.

The number *above* the line is called the ***numerator*** and shews how many of those equal parts are expressed by the fraction.

Thus $\frac{3}{4}$ means that an integer, or some whole quantity, is to be divided into 4 *equal* parts, and that 3 of those parts are expressed by the fraction $\frac{3}{4}$.

It may here also be observed, that the numerator of a fraction divided by the denominator will give the value of that fraction, and that every fraction denotes a division of its numerator by its denominator; thus $\frac{3}{4}$ denotes that the numerator 3 is to be divided by the denominator 4; it therefore appears that $\frac{3}{4}$ of 1 is the same thing with $\frac{1}{4}$ of 3.

To make this more plain; in the fraction $\frac{3}{4}$ let the integer be 1 lb. of sugar. Divide the 1 lb. or 16 oz. into 4 equal parts, and each part will be 4 oz., and 3 of these parts will be 12 oz.;—again, divide 3 lbs. or 48 oz. into 4 equal parts, and each part will be 12 oz.

It clearly then appears that $\frac{3}{4}$ of 1 lb. of sugar and $\frac{1}{4}$ of 3 lb. are the same thing.

Vulgar Fractions are of *four* kinds, simple, compound, mixed, and complex.

1. A *Simple Fraction* is one which has for its numerator and denominator whole numbers.

Simple Fractions are subdivided into Proper and Improper.

A Proper Fraction is one whose numerator is *less* than its denominator, as, $\frac{3}{4}$, $\frac{5}{9}$, $\frac{11}{13}$, &c.

An Improper Fraction is one whose numerator is *equal to*, or, *greater* than, its denominator, as, $\frac{5}{5}$, $\frac{4}{3}$, $\frac{9}{5}$, $\frac{13}{11}$, &c.

2. A *Compound Fraction* is a fraction *of* a fraction, as, $\frac{3}{4}$ of $\frac{5}{9}$ ( where $\frac{5}{9}$ is the quantity of which $\frac{3}{4}$ is to be taken,)—$\frac{3}{4}$ of $\frac{5}{9}$ of $\frac{11}{13}$, &c.

3. A *Mixed Fraction*, or *mixed number*, consists of a whole number and a fraction, as, $4\frac{5}{9}$, ( which signifies 4 integers and $\frac{5}{9}$ of an integer, as, £4 and $\frac{5}{9}$ of £1 ; 4 yards and $\frac{5}{9}$ of 1 yard ; &c., &c. according as the integer is £1, 1 yard, &c., &c.)

4. A complex fraction is one which has a fraction, or mixed number for its numerator or denominator, or for both, as

$$\frac{\frac{3}{4}}{9},\ \frac{9}{\frac{3}{4}},\ \frac{\frac{3}{4}}{\frac{5}{9}},\ \frac{4\frac{5}{9}}{7},\ \frac{15\frac{1}{2}}{17\frac{1}{4}},\ \&c.$$

Any whole number may be converted into a fraction by placing an unit (1) beneath it for a denominator, as, $\frac{5}{1}$ is the same as 5 ; $\frac{6}{1}$ as 6 ; &c.

When the numerator and denominator of a fraction are the same, the value of that fraction is 1, for as many parts are expressed by the fraction as the integer was divided into, thus $\frac{5}{5}$, $\frac{6}{6}$, $\frac{13}{13}$, &c. are all equal to 1.

When *both* the numerator and denominator of a fraction are multiplied or divided by the same number, the value of the fraction is not altered ; thus, $\frac{1}{2}$ is the same as $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, &c. where both numerator and denominator are multiplied by 2, 3, 4, &c. respectively ; and conversely $\frac{4}{8}$, $\frac{3}{6}$, $\frac{2}{4}$ are each of them equal to $\frac{1}{2}$.

The numerator and denominator of a fraction are frequently called *Terms*.

## REDUCTION OF VULGAR FRACTIONS.

**Reduction of Vulgar Fractions** teaches us to change the form, without changing the value, of a fraction.

Reduction of Vulgar Fractions may be divided into *ten* cases.

**Case 1.** To reduce a fraction to its lowest terms.

**Rule.** Divide the greater term of the fraction by the less, and that divisor by the remainder, and so on, dividing each divisor by the remainder till nothing remains; then divide both the numerator and denominator by the *last* divisor, which is called the greatest common measure, and the fraction will be reduced to its lowest terms.

OBSERVATIONS (1). The *greatest common measure* of any two or more quantities is the greatest number that will divide *both* or *all* of them without a remainder: thus 12 is the greatest common measure of 12, 24, 36, and 48.

(2). When the greatest common measure is 1, the fraction is in its lowest terms.

### EXAMPLES.

1. Reduce $\frac{144}{540}$ to its lowest terms.

```
144) 540 (3
     432
     ---
     108) 144 (1
          108
          ---
           36) 108 (3
               108
               ===
```

$\frac{144}{540}$ = 36) $\frac{144}{540}$ (= $\frac{4}{15}$ *Answer.*

The numerator and denominator being both divided by 36 the value of the fraction $\frac{144}{540}$ is not altered by being reduced to its lowest terms, $\frac{4}{15}$.

2. Reduce $\frac{102}{336}$ to its lowest terms.
3. Reduce $\frac{345}{1745}$ to its lowest terms.
4. Reduce $\frac{7568}{9504}$ to its lowest terms.
5. Reduce $\frac{1369}{1665}$ to its lowest terms.
6. Reduce $\frac{17810}{27707}$ to its lowest terms.

CASE 2. To reduce a mixed number to an improper fraction of equal value.

**Rule.** Multiply the whole quantity by the denominator of the fractional part, and add to their product the numerator of the fractional part, this sum placed over the denominator will give the improper fraction required.

### Examples.

1. Reduce $4\frac{5}{9}$ to an improper fraction.

$$4\tfrac{5}{9} = \frac{4 \times 9 + 5}{9} = \frac{41}{9} \quad \textit{Answer.}$$

$$\begin{array}{rl}
4 & \text{whole quantity} \\
9 & \text{denominator of fractional part} \\
\hline
36 & \text{product} \\
5 & \text{numerator of fractional part} \\
\hline
41 & \text{sum.} \\
\hline\hline
\end{array}$$

2. Reduce $37\frac{3}{7}$ to an improper fraction.
3. Reduce $184\frac{5}{9}$ to an improper fraction.
4. Reduce $26\frac{3}{11}$ to an improper fraction.
5. Reduce $173\frac{49}{1635}$ to an improper fraction.
6. Reduce $83\frac{4}{999}$ to an improper fraction.

**Case 3.** To reduce an improper fraction to a whole or mixed number of equal value.

**Rule.** Divide the numerator by the denominator, the quotient will be the whole number, the remainder placed over the denominator will be the fractional part.

### Examples.

1. Reduce $\frac{41}{9}$ to a whole or mixed number.

$$\frac{41}{9} = 4\tfrac{5}{9} \quad \textit{Answer.}$$

$$\begin{array}{rl}
9)\ 41 & (4 \text{ whole number.} \\
36 & \\
\hline
5 & \\
\hline\hline
\end{array}$$

2. Reduce $\frac{716}{47}$ to a whole or mixed number.
3. Reduce $\frac{266}{38}$ to a whole or mixed number.
4. Reduce $\frac{14327}{743}$ to a whole or mixed number.
5. Reduce $\frac{81435}{897}$ to a whole or mixed number.
6. Reduce $\frac{37049}{183}$ to a whole or mixed number.

CASE 4. To reduce a compound fraction to a simple fraction of equal value.

RULE. Reduce any of the fractions composing the compound fraction, that may require it, to simple fractions, then multiply all the numerators together, and all the denominators together, to form the numerator and denominator of the simple fraction required.

If, when the numerators and denominators are placed in order with the sign of multiplication (×) between them, it appears, that the same number occurs both in the numerator and denominator, it may be struck out of each; also if any number in the numerator, and any number in the denominator can be divided by any the same number without a remainder, they may be so divided, and their quotients used instead of them; this operation is called *cancelling*.

OBSERVATIONS (1). Since, in cancelling, both the numerator and denominator are divided by the same number, the value of the fraction is not altered.

(2). All numbers ending in 5 and 0 are divisible by 5.

(3). All numbers ending in 0 are divisible by 10.

(4). All even numbers are divisible by 2. *Even numbers* are those which end in 2, 4, 6, 8, 0.

(5). By cancelling the fraction is reduced to its *lowest* terms.

## EXAMPLES.

1. Reduce $\frac{3}{5}$ of $\frac{10}{11}$ of $3\frac{4}{9}$ of $5\frac{1}{9}$ of $\frac{3}{23}$ of 27 to a simple fraction.

$$3\tfrac{4}{9} = \frac{3\times9+4}{9} = \frac{31}{9}$$

$$5\tfrac{1}{9} = \frac{5\times9+1}{9} = \frac{46}{9}$$

$$27 = \frac{27}{1}$$

$$\tfrac{3}{5}\text{ of }\tfrac{10}{11}\text{ of }3\tfrac{4}{9}\text{ of }5\tfrac{1}{9}\text{ of }\tfrac{3}{23}\text{ of }27 = \tfrac{3}{5}\text{ of }\tfrac{10}{11}\text{ of }\tfrac{31}{9}\text{ of }\tfrac{46}{9}\text{ of }\tfrac{3}{23}\text{ of }\tfrac{27}{1}$$

$$\text{....................} = \frac{3\times\overset{2}{10}\times31\times\overset{2}{46}\times\overset{1}{3}\times\overset{\overset{1}{3}}{27}}{\underset{1}{5}\times11\times\underset{\underset{1}{3}}{9}\times\underset{1}{9}\times\underset{1}{23}\times1}$$

$$\text{....................} = \frac{3\times2\times31\times2}{11}$$

$$\text{....................} = \frac{372}{11}\ \textit{Answer.}$$

In cancelling the preceding example it appears that
10 in the num. and 5 in the den. will both divide by 5
46..............23 ...... .................. 23
3..... ........ 9 ......................... 3
27.............. 9 ......................... 9
3.............. 3 ......................... 3

2. Reduce $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{6}{13}$ of $\frac{14}{15}$ of 26 to a simple fraction.

3. Reduce $\frac{1}{2}$ of $\frac{37}{4}$ of $\frac{8}{74}$ of $\frac{2}{27}$ of $\frac{54}{7}$ of $\frac{14}{23}$ to a simple fraction.

4. Reduce $\frac{3}{7}$ of $\frac{63}{108}$ of $\frac{12}{27}$ of $\frac{9}{13}$ of $\frac{26}{29}$ of $\frac{29}{30}$ to a simple fraction.

5. Reduce $8\frac{2}{7}$ of $6\frac{2}{9}$ of $3\frac{3}{8}$ of $2\frac{9}{14}$ to a simple fraction.

6. Reduce $7\frac{4}{5}$ of $13\frac{8}{9}$ of $4\frac{4}{11}$ of $8\frac{5}{9}$ of 13 to a simple fraction.

Since Multiplication and Division of Vulgar Fractions are merely an application of this case, and since the remaining cases of Reduction are entirely dependent upon these two rules, they are here inserted.

## MULTIPLICATION OF VULGAR FRACTIONS.

**Rule.** Reduce the fractions, when necessary, to simple ones. Multiply all the numerators together for the numerator of the product, and all the denominators together for the denominator of the product. *Cancel* where it can be done, in order that the product may be in its *lowest terms*.

### Examples.

1. Multiply $3\frac{5}{8}$, $\frac{3}{5}$ of $\frac{9}{10}$, 14, and $\frac{5}{9}$.

$$3\tfrac{5}{8}=\frac{3\times 8+5}{8}=\frac{29}{8}$$

$$\tfrac{3}{5}\text{ of }\tfrac{9}{10}=\frac{3\times 9}{5\times 10}=\frac{27}{50}$$

$$3\tfrac{5}{8}\times\tfrac{3}{5}\text{ of }\tfrac{9}{10}\times 14\times\tfrac{5}{9}=\frac{29}{\underset{4}{8}}\times\frac{\overset{3}{27}}{\underset{10}{50}}\times\frac{\overset{7}{14}}{1}\times\frac{\overset{1}{5}}{\underset{1}{9}}=\frac{609}{40}=15\tfrac{9}{40}.$$

*Ans.*

2. Multiply $4\frac{1}{3}$, $\frac{3}{5}$ of $5\frac{1}{6}$, $7\frac{1}{4}$, and 15 together.

3. Multiply $16\frac{1}{5}$, $14\frac{1}{9}$, $\frac{1}{3}$ of $\frac{6}{11}$ of $2\frac{4}{9}$, and $\frac{1}{2}$ together

4. Multiply $2\frac{5}{6}$, $\frac{4}{5}$ of $3\frac{1}{3}$ of 7, 12, and $4\frac{1}{7}$ together.

5. Multiply $\frac{3}{8}$ of $\frac{4}{9}$ of $\frac{7}{12}$, $13\frac{1}{5}$, 43, and $\frac{2}{3}$ of $\frac{1}{4}$ of 15 together.

6. Multiply 18, $\frac{2}{7}$ of $\frac{3}{4}$ of $2\frac{4}{5}$, $6\frac{5}{6}$, and $13\frac{3}{8}$ together.

## DIVISION OF VULGAR FRACTIONS.

**Rule.** Reduce the fractions, when necessary, to simple ones. *Invert the divisor* and proceed as in Multiplication.

### Examples.

1. Divide $4\frac{3}{8}$ by $5\frac{1}{4}$.

$$4\frac{3}{8} = \frac{4\times8+3}{8} = \frac{35}{8}$$

$$5\frac{1}{4} = \frac{5\times4+1}{4} = \frac{21}{4}$$

$$4\frac{3}{8} \div 5\frac{1}{4} = \frac{35}{8} \div \frac{21}{4} = \frac{\overset{5}{35}}{\underset{2}{8}} \times \frac{\overset{1}{4}}{\underset{3}{21}} = \frac{5}{6} \text{ \textit{Answer.}}$$

2. Divide $\frac{17}{23}$ by $\frac{3}{7}$.
3. Divide $\frac{15}{49}$ by $\frac{3}{7}$.
4. Divide $\frac{6}{13}$ by 9.
5. Divide $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{4}{11}$ by $\frac{2}{3}$ of $\frac{5}{9}$ of $\frac{3}{8}$.
6. Divide $2\frac{7}{9}$ by $4\frac{1}{3}$.
7. Divide $4\frac{5}{6}$ by $6\frac{5}{12}$.
8. Divide $\frac{2}{7}$ of $\frac{3}{5}$ of $5\frac{5}{6}$ by $8\frac{1}{4}$.
9. Divide 14 by $3\frac{5}{9}$.

**Case 5.** To reduce a complex fraction to a simple fraction of equal value.

**Rule.** Divide the numerator by the denominator as in Division of Fractions.

### Examples.

1. Reduce $\dfrac{8\frac{5}{9}}{9\frac{1}{5}}$ to a simple fraction.

$$\frac{8\frac{5}{9}}{9\frac{1}{5}}=8\tfrac{5}{9}\div 9\tfrac{1}{5}=\tfrac{77}{9}\div\tfrac{46}{5}=\tfrac{77}{9}\times\tfrac{5}{46}=\tfrac{385}{414}$$ *Answer.*

$$8\tfrac{5}{9}=\frac{8\times 9+5}{9}=\frac{77}{9}$$

$$9\tfrac{1}{5}=\frac{9\times 5+1}{5}=\frac{46}{5}$$

2. Reduce $\dfrac{6}{\frac{2}{5}}$ to a simple fraction.

3. Reduce $\dfrac{\frac{4}{9}}{11}$ to a simple fraction.

4. Reduce $\dfrac{\frac{3}{7}}{\frac{2}{3}}$ to a simple fraction.

5. Reduce $\dfrac{17\frac{1}{5}}{19\frac{1}{4}}$ to a simple fraction.

6. Reduce $\dfrac{4\frac{3}{7}}{5\frac{4}{9}}$ to a simple fraction.

7. Reduce $\dfrac{15\frac{1}{8}}{43\frac{5}{11}}$ to a simple fraction.

CASE 6. To reduce fractions of different denominators to others of equal value, having a common denominator.

RULE. Reduce such of the fractions as require it to simple fractions. Multiply *each* numerator by all the denominators *except its own* for a new numerator, and *all* the denominators together for a common denominator.

## EXAMPLES.

1. Reduce $\frac{3}{4}$, $\frac{5}{9}$, $1\frac{1}{5}$ and $\frac{1}{7}$ to fractions of equal value having a common denominator.

$$1\frac{1}{5}=\frac{1\times5+1}{5}=\frac{6}{5}$$

then $\frac{3}{4}$, $\frac{5}{9}$, $\frac{6}{5}$, $\frac{1}{7}$ are the simple fractions to be reduced.

$$\frac{3}{4}=\frac{3}{4}\times\frac{9}{9}\times\frac{5}{5}\times\frac{7}{7}=\frac{935}{1260}$$
$$\frac{5}{9}=\frac{5}{9}\times\frac{4}{4}\times\frac{5}{5}\times\frac{7}{7}=\frac{700}{1260}$$
$$\frac{6}{5}=\frac{6}{5}\times\frac{4}{4}\times\frac{9}{9}\times\frac{7}{7}=\frac{1512}{1260}$$
$$\frac{1}{7}=\frac{1}{7}\times\frac{4}{4}\times\frac{9}{9}\times\frac{5}{5}=\frac{180}{1260}$$

*Answer* $\frac{935}{1260}$, $\frac{700}{1260}$, $\frac{1512}{1260}$, $\frac{180}{1260}$.

It is evident from the preceding example that the value of the fractions is not altered, as both the numerator and the denominator of each fraction is multiplied by the *same* number:—for $\frac{3}{4}$ is multiplied by $\frac{9}{9}$, $\frac{5}{5}$ and $\frac{7}{7}$ successively, each of which fractions is equal to 1.

2. Reduce $\frac{1}{7}$, $\frac{4}{9}$, $\frac{3}{5}$ and $\frac{5}{4}$ to fractions having a common denominator.

3. Reduce $3\frac{1}{4}$, $\frac{6}{7}$, $4\frac{1}{5}$ and 17 to fractions having a common denominator.

4. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{8}{9}$, $7\frac{1}{6}$, and $\frac{1}{7}$ of $2\frac{7}{11}$ to fractions having a common denominator.

5. Reduce $\frac{4}{9}$ of $5\frac{5}{8}$, $\frac{6}{13}$, $11\frac{5}{9}$, and 4 to fractions having a common denominator.

6. Reduce $\frac{3}{4}$ of $4\frac{1}{4}$, $\frac{2}{3}$ of 5, $\frac{3}{8}$ of 9, and $\frac{1}{5}$ of 3 to fractions having a common denominator.

**CASE 7.** To reduce fractions of different denominators to others, of equal value, having the least common denominator.

**Rule.** Reduce such of the fractions as require it to simple fractions. Arrange the denominators in one line, find what number will divide any two or more of the denominators without a remainder, divide by such number; place the quotients and undivided numbers in a line beneath; divide this line as before, and so on, till there are not two numbers that can be divided. The products of the divisors, quotients, and undivided numbers in the last line will give the least common denominator. Then multiply *both* terms of every one of the fractions by this least common denominator, cancelling the denominator of every one of the given fractions.

**Observations** (1). The least common denominator is likewise called the least common multiple of the numbers forming the denominators of the fractions.

(2). The least common multiple of any two or more numbers is the *least* number that can be divided by *each* of such numbers without a remainder.

The preceding rule will be better understood from the following example.

### Examples.

1. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{9}$, $\frac{2}{3}$, and $\frac{5}{12}$ to fractions, of equal value, having the least common denominator.

$$\begin{array}{r|l} 3) & 2\ .\ 4\ .\ 9\ .\ 3\ .\ 12\ . \\ \hline 2) & 2\ .\ 4\ .\ 3\ .\ 1\ .\ 4\ . \\ \hline 2) & 1\ .\ 2\ .\ 3\ .\ 1\ .\ 2\ . \\ \hline & 1\ .\ 1\ .\ 3\ .\ 1\ .\ 1\ . \end{array}$$

$3\times2\times2\times3=36$ least common denominator.

$$\frac{1}{2}=\frac{1}{\underset{1}{2}}\times\frac{\overset{18}{36}}{36}=\frac{18}{36}$$

$$\frac{3}{4}=\frac{3}{\underset{1}{4}}\times\frac{\overset{9}{36}}{36}=\frac{27}{36}$$

$$\frac{5}{9}=\frac{5}{\underset{1}{9}}\times\frac{\overset{4}{36}}{36}=\frac{20}{36}$$

$$\frac{2}{3}=\frac{2}{\underset{1.}{3}}\times\frac{\overset{12}{36}}{36}=\frac{24}{36}$$

$$\frac{5}{12}=\frac{5}{\underset{1}{12}}\times\frac{\overset{3}{36}}{36}=\frac{15}{36}$$

*Answer* $\frac{18}{36}$, $\frac{27}{36}$, $\frac{20}{36}$, $\frac{24}{36}$, $\frac{15}{36}$.

**Explanation.** In the preceding example, 36, being the least common multiple of 2 . 4 . 9 . 3 . 12, is divisible by each of those numbers without a remainder, and is the least number so divisible.

2. Reduce $\frac{3}{8}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{9}$ and $\frac{1}{6}$ to fractions, of equal value, having the least common denominator.
3. Reduce $4\frac{1}{3}$, $3\frac{5}{12}$, $7\frac{1}{4}$ and $\frac{3}{5}$ of $\frac{5}{18}$ to fractions, of equal value, having the least common denominator.
4. Reduce $\frac{1}{7}$, $\frac{5}{6}$, $\frac{3}{14}$, and $\frac{1}{2}$ of $\frac{4}{21}$ to fractions, of equal value, having the least common denominator.
5. Reduce $\frac{2}{21}$, $\frac{4}{15}$, $2\frac{2}{3}$, and $1\frac{5}{63}$ to fractions, of equal value, having the least common denominator.
6. Reduce $\frac{2}{9}$, $\frac{1}{4}$ of $2\frac{1}{3}$, $3\frac{5}{8}$, $4\frac{1}{6}$ and $\frac{7}{24}$ to fractions, of equal value, having the least common denominator.

**Case 8.** To reduce a fraction of one denomination to the fraction of another denomination of equal value.

**Rule (1).** When the reduction is from a higher denomination to a lower; then, *multiply* the fraction by as many of the next lower denomination as make one of the higher, and so on, till the required denomination is obtained.

(2). When the reduction is from a lower denomination to a higher; then, *divide* the fraction by as many of the *same* denomination as make one of the next higher, and so on, till the required denomination is obtained.

**Observation.** This rule is exactly the same as that for Simple Reduction.

## EXAMPLES.

1. Reduce $\frac{3}{7}$ of a pound to the fraction of a farthing.

$$£\tfrac{3}{7}=\tfrac{3}{7}\times\tfrac{20}{1}\times\tfrac{12}{1}\times\tfrac{4}{1}=\tfrac{2880}{7} \text{ of a farthing.}$$

EXPLANATION. In the preceding example a quantity of a higher denomination is to be reduced to a lower, namely, a fraction of a pound to the fraction of a farthing; and as in whole numbers we reduce pounds into farthings by multiplying the pounds by 20 to reduce them into shillings, the shillings by 12 to reduce them into pence, and the pence by 4 to reduce them into farthings, so in the example given the same operation is performed.

2. Reduce $\frac{3}{7}$ of a farthing to the fraction of a pound.

$$\tfrac{3}{7}=\tfrac{\overset{1}{3}}{7}\times\tfrac{1}{4}\times\tfrac{1}{\underset{4}{12}}\times\tfrac{1}{20}=\tfrac{1}{2240} \text{ of a pound.}$$

EXPLANATION. Here a quantity of a lower denomination is to be reduced to a higher, namely, a fraction of a farthing to the fraction of a pound: the fraction is therefore divided by 4, 12, and 20, as in whole numbers; or what is the same thing, multiplied by $\frac{1}{4}$, $\frac{1}{12}$, and $\frac{1}{20}$, which are the reciprocals of 4, 12, and 20.

3. Reduce $\frac{4}{9}$ of a pound to the fraction of a penny.
4. Reduce $\frac{3}{11}$ of a farthing to the fraction of a pound.
5. Reduce $\frac{13}{19}$ of a pound (troy) to the fraction of a grain.
6. Reduce $\frac{6}{13}$ of a ton to the fraction of a pound (avoirdupoise).
7. Reduce $\frac{17}{25}$ of an English ell to the fraction of a nail.
8. Reduce $\frac{35}{48}$ of a mile to the fraction of yard.
9. Reduce $\frac{9}{17}$ of a day to the fraction of a year.

10. Reduce $\frac{3}{11}$ of a crown to the fraction of a guinea.

11. Reduce $\frac{5}{9}$ of a pound to the fraction of a guinea.

12. Reduce $\frac{3}{4}$ of half-a-crown to the fraction of a shilling.

**Case 9.** To reduce a compound quantity to a fraction of any denomination.

**Rule.** Reduce the given quantity to the lowest denomination mentioned in it: if the quantity so reduced be a whole number, convert it into a fraction by placing 1 beneath it for a denominator, and reduce the fraction to the required denomination by the preceding case.

## Examples.

1. Reduce 7*s*. 11$\frac{3}{4}$*d*. to the fraction of a pound.

$$7s.\ 11\tfrac{3}{4}d. = 95\tfrac{3}{4}d. = \frac{95 \times 4 + 3}{4} = \tfrac{383}{4}d.$$

$$\tfrac{383}{4}d. = \tfrac{383}{4} \times \tfrac{1}{12} \times \tfrac{1}{20} = \tfrac{383}{960} \text{ of a pound.}$$

2. Reduce 8*s*. 6$\frac{1}{4}$*d*. to the fraction of a pound.

3. Reduce 2*s*. 7$\frac{1}{9}$*d*. to the fraction of a pound.

4. Reduce 5 oz. 16$\frac{1}{3}$ dwt. to the fraction of a pound (troy).

5. Reduce 3 qrs. 2$\frac{2}{7}$ n. to the fraction of a French ell.

6. Reduce 17 gal. 1$\frac{3}{11}$ qt. to the fraction of a pipe of wine.

7. Reduce 3 bush. 2$\frac{4}{5}$ pecks to the fraction of a quarter of corn.

8. What part of a pound is 17*s*. 4$\frac{3}{4}$*d*.?

9. What part of a pound (avoirdupoise) is 3 ounces 13$\frac{4}{9}$ dr.?

10. What part of an English ell is 2 qr. 2$\frac{1}{6}$ n.?

CASE 10. To find the value of a fraction in the known parts of the integer.

RULE. Multiply the fraction by as many of the next lower denomination as make *one* of the higher; divide that product by the denominator; if there be any remainder, multiply it by the parts of the next inferior denomination, divide as before, and so on: the quotients placed in order will be the value of the fraction required.

OBSERVATION (1). This process is merely an application of the Rule for Compound Division.

(2). If the integer is £1, the known parts of the integer are shillings, pence, and farthings; if 1 cwt., then quarters, pounds, ounces, and drachms; &c., &c.

## EXAMPLES.

1. Find the value of $\frac{3}{7}$ of a pound.

$$£\tfrac{3}{7}=\tfrac{3}{7}\times\tfrac{20}{1}=\tfrac{60}{7}s.=8s.\ 6\tfrac{3}{4}\ \tfrac{3}{7}d.\ \textit{Answer.}$$

$$\begin{array}{l}
\quad\ s. \\
7)\ \underline{60} \\
\quad\ 8\ .\ 4 \\
\qquad\ \underline{12} \\
\quad 7)\ \underline{48} \\
\qquad\ 6\ .\ 6 \\
\qquad\quad\ \underline{4} \\
\qquad 7)\ \underline{24} \\
\qquad\quad\ \underline{\underline{3\ \tfrac{3}{7}}}
\end{array}$$

2. Find the value of $\frac{7}{11}$ of a pound.
3. Find the value of $\frac{4}{7}$ of a crown.

4. Find the value of $\frac{1}{8}$ of a guinea.
5. Find the value of $\frac{3}{5}$ of a pound (troy).
6. Find the value of $\frac{3}{8}$ of 19 yards of cloth.
7. Find the value of $\frac{4}{5}$ of $\frac{3}{11}$ of $\frac{7}{12}$ of a ton of iron.
8. Find the value of $\frac{5}{7}$ of 30 pounds sterling.
9. Find the value of $\frac{6}{11}$ of a pipe of wine.
10. Find the value of $\frac{13}{25}$ of a year.
11. Find the value of $\frac{3}{7}$ of a league.
12. Find the value of $\frac{2}{9}$ of an ell English.
13. Find the value of $\frac{3}{7}$ of $\frac{4}{9}$ of 7 cwt. 2 qr. 13 lb.
14. Find the value of $\frac{3}{5}$ of $\frac{1}{7}$ of £53 10*s*.
15. Find the value of $\frac{4}{9}$ of $\frac{2}{7}$ of 28 English ells 3 quarters 2 nails.
16. What will be the value of $\frac{6}{11}$ of an estate worth £12759?

## ADDITION OF VULGAR FRACTIONS.

ADDITION OF VULGAR FRACTIONS may be divided into *two* cases.

CASE 1. Where quantities of the *same* denomination are to be added.

CASE 2. Where quantities of *different* denominations are to be added.

CASE 1. RULE. Reduce the Fractions, when necessary, to simple fractions, and fractions having different denominators to others having the *least* common denominator. Add the *numerators* together, and place their sum over the common denominator, the result will be the *sum* of the fractions required.

OBSERVATION. It is necessary to bring the fractions to a common denominator, as quantities of *like* magnitude only can be added together.

## EXAMPLES.

1. Add $\frac{1}{3}$ of $\frac{3}{5}$, $\frac{3}{4}$ of 17, and $4\frac{5}{12}$ together.

$$\frac{1}{3}\text{ of }\frac{3}{5}=\frac{1\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}\times5}=\frac{1}{5}$$

$$\frac{3}{4}\text{ of }17=\frac{3\times17}{4\times1}=\frac{51}{4}$$

$$4\frac{5}{12}=\frac{4\times12+5}{12}=\frac{53}{12}$$

$$\frac{1}{5}+\frac{51}{4}+\frac{53}{12}=\frac{12}{60}+\frac{765}{60}+\frac{265}{60}=\frac{1042}{60}=17\frac{22}{60}=17\frac{11}{30}\ \textit{An.}$$

$$\begin{array}{r|l} 4) & \underline{5\ .\ 4\ .\ 12} \\ & 5\ .\ 1\ .\ \ 3 \end{array}$$

$4\times5\times3=60$ least common denominator.

$$\frac{1}{5}=\frac{1}{\underset{1}{\cancel{5}}}\times\frac{\overset{12}{\cancel{60}}}{60}=\frac{12}{60}$$

$$\frac{51}{4}=\frac{51}{\underset{1}{\cancel{4}}}\times\frac{\overset{15}{\cancel{60}}}{60}=\frac{765}{60}$$

$$\frac{53}{12}=\frac{53}{\underset{1}{\cancel{12}}}\times\frac{\overset{5}{\cancel{60}}}{60}=\frac{265}{60}$$

When large mixed numbers or whole numbers and fractions are to be added, it will be better to add the fractional parts *separately*, and *their* sum added to the sum of the *whole numbers* will give the whole sum required.

2. Add $73\frac{1}{3}$, $49\frac{5}{6}$, and $3\frac{1}{4}$ together.

$$\begin{array}{l} 73\frac{1}{3}+49\frac{5}{6}+3\frac{1}{4}=73+49+3+\frac{1}{3}+\frac{5}{6}+\frac{1}{4} \\ \ldots\ldots\ldots\ldots=125+1\frac{5}{12} \\ \ldots\ldots\ldots\ldots=126\frac{5}{12} \quad \textit{Answer.} \end{array}$$

$$\frac{1}{3}+\frac{5}{6}+\frac{1}{4}=\frac{4}{12}+\frac{10}{12}+\frac{3}{12}=\frac{17}{12}=1\frac{5}{12}$$

$$\begin{array}{r|l} 3) & 3\ .\ 6\ .\ 4 \\ \hline 2) & 1\ .\ 2\ .\ 4 \\ \hline & 1\ .\ 1\ .\ 2 \\ \hline\hline \end{array}$$

$3\times2\times2=12$ least common denominator.

$$\frac{1}{3}=\frac{1}{\underset{1}{3}}\times\frac{\overset{4}{12}}{12}=\frac{4}{12}$$

$$\frac{5}{6}=\frac{5}{\underset{1}{6}}\times\frac{\overset{2}{12}}{12}=\frac{10}{12}$$

$$\frac{1}{4}=\frac{1}{\underset{1}{4}}\times\frac{\overset{3}{12}}{12}=\frac{3}{12}$$

3. Add $\frac{1}{7}$, $3\frac{1}{4}$, $\frac{4}{5}$, and $4\frac{1}{8}$ together.
4. Add $7\frac{1}{3}$, $49\frac{7}{11}$, $103\frac{2}{9}$, and 17 together.
5. Add $\frac{1}{3}$, $4\frac{1}{4}$, $3\frac{1}{6}$, and $7\frac{5}{12}$ together.
6. Add $\dfrac{5\frac{1}{4}}{3}$, $\dfrac{4\frac{1}{3}}{7\frac{1}{5}}$, and $\dfrac{7}{\frac{1}{9}}$ together.
7. Add $27\frac{1}{5}$, $328\frac{1}{9}$, $14\frac{1}{3}$, and $\frac{6}{15}$ together.
8. Add $\frac{1}{7}$, $3\frac{4}{9}$, $\dfrac{8\frac{2}{3}}{5\frac{1}{5}}$, and $\dfrac{4\frac{2}{7}}{3}$ together.

CASE 2. RULE. Find the value of the quantities in the known parts of their integers, and add these values as in Compound Addition.

### EXAMPLES.

1. Add $\frac{1}{3}$ of a pound, $\frac{1}{7}$ of 15 pounds, $\frac{4}{9}$ of $\frac{3}{5}$ of $\frac{5}{8}$ of a shilling together.

$$\hphantom{\tfrac{1}{3}=\tfrac{1}{3}\times\tfrac{20}{1}=\tfrac{20}{3}=\ldots\ldots\ldots\ldots\ldots\ldots} \pounds \quad s. \quad d.$$

$$\overset{\pounds}{\tfrac{1}{3}}=\tfrac{1}{3}\times\tfrac{20}{1}=\overset{s.}{\tfrac{20}{3}}=\ldots\ldots\ldots\ldots\ldots\ldots \quad 0 \quad 6 \quad 8$$

$$\tfrac{1}{7}\text{ of }\overset{\pounds}{15}=\tfrac{1}{7}\times\tfrac{15}{1}=\overset{\pounds}{\tfrac{15}{7}}=\ldots\ldots\ldots\ldots \quad 2 \quad 2 \quad 10\tfrac{1}{4}.\tfrac{1}{7}$$

$$\tfrac{4}{9}\text{ of }\tfrac{3}{5}\text{ of }\tfrac{5}{8}=\frac{\overset{1}{4}\times\overset{1}{3}\times\overset{1}{5}}{\underset{3}{9}\times\underset{1}{5}\times\underset{2}{8}}=\overset{s.}{\tfrac{1}{6}}=\frac{1}{\underset{1}{6}}\times\frac{\overset{2}{12}}{1}= \quad 0 \quad 0 \quad 2$$

$$\textit{Answer} \quad \pounds 2 \quad 9 \quad 8\tfrac{1}{4}.\tfrac{1}{7}$$

2. Add $\frac{2}{5}$ of a pound, $\frac{3}{9}$ of a shilling, and $\frac{1}{4}$ of $\frac{3}{5}$ of a penny together.

3. Add $\frac{2}{7}$ of a crown, $\frac{5}{9}$ of a pound, and $\frac{3}{8}$ of a guinea together.

4. What is the sum of $\frac{3}{4}$ of £3 14*s*., $\frac{2}{7}$ of £3 10*s*., and $\frac{1}{3}$ of £4 9s. 6*d*.?

5. What is the sum of $\frac{4}{9}$ of a ton, $\frac{1}{3}$ of a cwt., and $\frac{5}{6}$ of a pound (avoirdupoise)?

6. What is the sum of $\frac{2}{5}$ of a yard, $\frac{4}{7}$ of an English ell, and $\frac{1}{2}$ of a Flemish ell?

7. What is the sum of $\frac{2}{3}$ of a mile, $\frac{4}{9}$ of a league, and $\frac{1}{12}$ of a furlong?

8. What is the sum of $\frac{3}{4}$ of a tun of wine, $\frac{1}{5}$ of a hogshead, and $\frac{4}{7}$ of a gallon?

## SUBTRACTION OF VULGAR FRACTIONS.

SUBTRACTION OF VULGAR FRACTIONS may, like Addition, be divided into *two* cases.

**Case 1.** Where quantities of the *same* denomination are to be subtracted.

**Case 2.** Where quantities of *different* denominations are to be subtracted.

**Case 1. Rule.** Prepare the fractions, as in Addition, by reducing them to other fractions having the least common denominator. Then the difference of the numerators placed over the common denominator will be the difference of the fractions required.

## EXAMPLES.

1. From $\frac{3}{5}$ take $\frac{5}{11}$.

$$\frac{3}{5}=\frac{3}{5}\times\frac{11}{11}=\frac{33}{55}$$
$$\frac{5}{11}=\frac{5}{11}\times\frac{5}{5}=\frac{25}{55}$$
$$\frac{3}{5}-\frac{5}{11}=\frac{33}{55}-\frac{25}{55}=\frac{8}{55}\quad\textit{Answer.}$$

2. Subtract $7\frac{1}{3}$ from $\frac{3}{4}$ of $\frac{5}{9}$ of 73.

$$\frac{3}{4}\text{ of }\frac{5}{9}\text{ of }73=\frac{\overset{1}{3}\times5\times73}{4\times\underset{3}{9}\times1}=\frac{365}{12}=30\frac{5}{12}$$

$$30\frac{5}{12}-7\frac{1}{3}=30\frac{5}{12}-7\frac{4}{12}=23\frac{1}{12}\quad\textit{Answer.}$$

**Explanation.** In the preceding example, the quantities being mixed numbers, the fractional parts only were reduced to a common denominator.

3. From 275 take $7\frac{5}{9}$.

$$275=274\frac{9}{9}\text{ for }\frac{9}{9}=1$$
$$275-7\frac{5}{9}=274\frac{9}{9}-7\frac{5}{9}=267\frac{4}{9}\quad\textit{Answer.}$$

4. From $\frac{4}{9}$ take $\frac{3}{7}$.

5. What is the difference between $6\frac{1}{9}$ and $8\frac{3}{11}$?

6. What is the difference between $\frac{4\frac{1}{7}}{3\frac{5}{8}}$ and $\frac{2\frac{4}{9}}{7\frac{1}{6}}$?

7. What is the difference between $117\frac{2}{11}$ and $49\frac{5}{7}$?

8. What is the difference between $35\frac{1}{4}$ and $14\frac{2}{3}$?

9. Subtract $\frac{11}{47}$ from an unit.

**Case 2. Rule.** Find the value of the quantitie as in Addition of Fractions. Subtract the less fron the greater as in Compound Subtraction.

### Examples.

1. What is the difference between $\frac{1}{3}$ of 5 pound and $\frac{2}{9}$ of a guinea?

$$\begin{array}{lrrr}
\quad\;\text{£}\qquad\quad\;\text{£} & \text{£} & s. & d. \\
\frac{1}{3}\text{ of }5=\frac{1}{3}\times\frac{5}{1}=\frac{5}{3}=\ldots\ldots & 1 & 13 & 4 \\
\textit{guinea}\qquad\quad s. & & & \\
\frac{2}{9}=\frac{2}{\underset{3}{9}}\times\frac{\overset{7}{21}}{1}=\frac{14}{3}=\ldots\ldots\ldots & 0 & 4 & 8 \\
\hline
 & \text{£}1 & 8 & 8\ \textit{Ans.}
\end{array}$$

2. What is the difference between $\frac{2}{3}$ of a pound, and $\frac{3}{4}$ of a shilling?

3. What is the difference between $\frac{1}{7}$ of £4 3*s*., and $\frac{3}{5}$ of £7 9*s*?

4. What is the difference between $\frac{3}{8}$ of a pound (troy), and $\frac{1}{3}$ of an ounce?

5. What is the difference between $\frac{1}{6}$ of a ton, and $\frac{2}{3}$ of $\frac{1}{7}$ of a cwt.?

6. What is the difference between $\frac{3}{5}$ of $\frac{4}{9}$ of a yard, and $\frac{2}{7}$ of $\frac{1}{3}$ of an English ell?

7. What is the difference between $\frac{4}{7}$ of a league, and $\frac{5}{6}$ of a mile?

8. What is the difference between $\frac{6}{13}$ of $365\frac{1}{4}$ days, and $\frac{1}{2}$ of a week?

## RULE OF THREE IN VULGAR FRACTIONS.

**Rule.** State the question as in the common Rule of Three. Reduce such of the terms as require it to simple fractions, and the first and second terms to fractions of the same denomination. Multiply the second and third terms together, and divide by the first, the value of the resulting fraction must then be found, in order that the answer may be exhibited in its proper form.

### Examples.

1. If $\frac{3}{4}$ of a yard of muslin cost 2*s*. 7*d*. what will $27\frac{5}{8}$ ells cost?

$$\text{As } \overset{yd.}{\tfrac{3}{4}} : \overset{ell}{27\tfrac{5}{8}} :: \overset{s.\ d.}{2\ 7} : \text{number required.}$$

$$\frac{3}{4} = \frac{3}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{5} = \frac{3}{5}\ ell$$

$$\overset{ell}{27\tfrac{5}{8}} = \frac{27 \times 8 + 5}{8} = \frac{221}{8}\ ell$$

$$2s\ 7d. = 31d. = \frac{31}{1} \times \frac{1}{12} \times \frac{1}{20} = \frac{31}{240}\text{£}.$$

$$\text{Therefore. As } \overset{ell}{\tfrac{3}{5}} : \overset{ell}{\tfrac{221}{8}} :: \overset{\text{£}}{\tfrac{31}{240}} : \text{number required.}$$

$$\frac{221}{8} \times \frac{31}{240} \div \frac{3}{5} = \frac{221}{8} \times \frac{31}{\underset{48}{\cancel{240}}} \times \frac{\overset{1}{\cancel{5}}}{3} = \overset{\text{£}}{\frac{6851}{1152}} = \overset{\text{£}\ \ s.\ \ d.}{5\ 18\ 11\tfrac{1}{4}.\tfrac{1}{6}}\ An.$$

**Observation.** It will be found advantageous to state the question a second time, after the three terms have been reduced, as in the preceding example.

2. If $\frac{3}{7}$ of a lb. cost 8*s.* $3\frac{1}{4}$*d.* what will be the cost of 1 cwt.?

3. Bought 17 pieces of French merino, each containing $65\frac{3}{4}$ English ells at 9*s.* $10\frac{1}{2}$*d.* per yard. What was the value of the whole?

4. What quantity of carpet, that is $1\frac{3}{8}$ yard wide, will cover a floor that is 23 feet long and 16 feet wide?

5. If I possess a $\frac{1}{7}$ share of a ship, and sell $\frac{3}{4}$ of my share for £875, what is the value of the whole ship?

6. By the will of a relation, a person becomes possessed of $\frac{3}{4}$ of the landed property of the testator, which consists of $\frac{2}{3}$ of an estate, he, being in want of money, sells $\frac{4}{9}$ of his share for £1647 6*s.* 8*d.* What was the value of the whole estate?

7. Bought $4\frac{1}{4}$ pieces of silk each containing $37\frac{3}{8}$ ells English at 5*s.* $3\frac{1}{9}$*d.* per yard. What did the whole cost?

8. At $7\frac{3}{4}$*d.* per lb., what will be the cost of 11 hogsheads of sugar each weighing 4 cwt. 3 qrs. $15\frac{5}{8}$lb.?

9. A person who possessed a $\frac{3}{7}$ share of a lead mine, sold $\frac{4}{5}$ of his interest in it for £3761 17*s.* 6*d.*; what was the value of the whole mine?

## DECIMAL FRACTIONS.

DECIMAL FRACTIONS are those which have for their denominators 10, 100, 1000, &c., thus $\frac{3}{10}$, $\frac{17}{100}$, $\frac{4}{1000}$ &c. are decimal fractions.

*A Decimal Fraction* is expressed by writing the *numerator only*, with a point (·) before it towards the left hand; which numerator, when so written, must always consist of as many places of figures, as there were *ciphers* in the denominator; should there *not* be as many places of figures in the numerator as there were ciphers in the denominator, as many ciphers as will supply the deficiency must be added to the left hand of the numerator. Thus

$\frac{3}{10}$ is *written* ·3 and *read* decimal 3
$\frac{17}{100}$ ........ ·17 .............. 17
$\frac{4}{1000}$ ........ ·004 .............. 004
where ·3, ·17, ·004 are called *Decimals.*

Hence the *denominator* of a decimal fraction consists of an unit (1) with as many ciphers annexed as there are places of figures in the decimal.

Ciphers added to the right hand of decimals make *no* alteration in their value; for

·5 = $\frac{5}{10}$ = $\frac{1}{2}$
·50 = $\frac{50}{100}$ = $\frac{1}{2}$
·500 = $\frac{500}{1000}$ = $\frac{1}{2}$
·5000 = $\frac{5000}{10000}$ = $\frac{1}{2}$

where it is plain that ·5, ·50, ·500, ·5000, are all equal to $\frac{1}{2}$, and consequently to one another. Ciphers added to the left hand of decimals *decrease* their value in a tenfold proportion; for

.5 = $\frac{5}{10}$
·05 = $\frac{5}{100}$ = $\frac{5}{10} \div 10$
·005 = $\frac{5}{1000}$ = $\frac{5}{100} \div 10$
·0005 = $\frac{5}{10000}$ = $\frac{5}{1000} \div 10$

where it is plain that every successive decimal is th preceding divided by 10, or is ten times less than th preceding decimal; therefore, reckoning from the righ hand of the unit's place, in any decimal

the 1st fig. expresses so many tenths,
2nd ................ hundredths,
3rd ................ thousandths
4th ................ tenths of thousandths,
5th ................ hundredths of thousandths,
6th ................ millionths,
&c., &c., &c.,

thus in the decimal ·123456 reckoning from the left hand

the 1st figure expresses one tenth ($\frac{1}{10}$) or ·1,
2nd ............ two hundredths ($\frac{2}{100}$) or ·02,
3rd ............ three thousandths ($\frac{3}{1000}$) or ·003,
4th ............ four tenths of thousandths ($\frac{4}{10000}$) or ·0004,
5th ............ five hundredths of thousandths ($\frac{5}{100000}$) or ·00005
6th ............ six millionths ($\frac{6}{1000000}$) or ·000006.

From the above remarks we learn that as whole numbers increase in a tenfold proportion from the unit's place towards the left hand, so decimals decrease in a tenfold proportion from the unit's place towards the right hand.

A *mixed number* is composed of a whole number and a decimal, as, 7·315 or $7\frac{315}{1000}$.

A *recurring decimal* is that in which the same figure or series of figures are infinitely repeated, as, 3·16666, &c.; 7·5757, &c.

## ADDITION OF DECIMALS.

**Rule.** Place the quantities so that the decimal points may be under each other, or so that tenths may be under tenths, hundredths under hundredths, &c. Add as in whole numbers, and point off from the right hand of the sum as many places for decimals, as there were in the number having the greatest number of places of decimals.

The proof of this rule may be shewn, thus;

Add ·5, ·05, ·005 together.

$$\begin{array}{rl} \text{Now} & .5=\frac{5}{10} \\ & \cdot05=\frac{5}{100} \\ & \cdot005=\frac{5}{1000} \\ \text{therefore} & \cdot5+\cdot05+\cdot005=\frac{5}{10}+\frac{5}{100}+\frac{5}{1000} \\ & \ldots\ldots\ldots\ldots=\frac{500}{1000}+\frac{50}{1000}+\frac{5}{1000} \\ & \ldots\ldots\ldots\ldots=\frac{555}{1000} \\ & \ldots\ldots\ldots\ldots=\cdot555 \end{array}$$

which consists of as many places of decimals as there were in the number having the greatest number of decimals.

## Examples.

1. Add 7·31569, ·004, 39·1635, 475·75, ·0003 together.

$$\begin{array}{r@{\cdot}l} 7&31569 \\ &004 \\ 39&1635 \\ 475&75 \\ &0003 \\ \hline 522&23349 \end{array}$$

2. Add together 5·149, ·0003, 15·25, 369·8, 14, ·009.

3. Add together ·153, 7·146, ·0007, 1·004, ·137845.

4. Add together ·006, ·0095, ·4, 36·003, ·00796.

5. Add together 71·35, ·00825, 7·1276, 19, 32·1704.

## SUBTRACTION OF DECIMALS.

**Rule.** Place the quantities under each other, according to the value of their places, as in Addition ; subtract as in whole numbers, and point off, from the remainder, the decimal places as in Addition.

The proof of this rule may be shewn, thus ;

Subtract ·004 from ·5.

Now ·004 $=\frac{4}{1000}$
·5 $=\frac{5}{10}$
therefore ·5—·004 $=\frac{5}{10}-\frac{4}{1000}$
...... $=\frac{500}{1000}-\frac{4}{1000}$
...... $=\frac{496}{1000}$
...... =·496

which consists of as many places of decimals as there were in the number having the greater number of places of decimals.

### Examples.

1. From 27·15369 take ·000347.

$$\begin{array}{r} 27{\cdot}15369\phantom{0} \\ {\cdot}000347 \\ \hline 27{\cdot}153343 \\ \hline \hline \end{array}$$

2. From 39·27345 take 13·148276.
3. From 3·75145 take ·0375145.
4. From 1734·692 take ·1734692.
5. Subtract ·00475 from an unit.
6. Subtract 26·1437 from 100.

## MULTIPLICATION OF DECIMALS.

**Rule.** Multiply as in whole numbers, and, from the right hand of the product, point off as many places for decimals as there are decimal places both in the multiplier and multiplicand together.

**Observation.** If the number of places of figures in the product is less than the number of decimal places both in the multiplier and multiplicand, then, as many *ciphers* as will supply the deficiency *must* be added to the *left hand* of the product, to give the true product.

The proof of this rule may be shewn, thus;

Multiply ·04 by ·07.

$$\begin{array}{rrl} & \text{Now} \quad \cdot 04 & = \frac{4}{100} \\ & \text{and} \quad \cdot 07 & = \frac{7}{100} \\ \text{therefore} & \cdot 04 \times \cdot 07 & = \frac{4}{100} \times \frac{7}{100} \\ & \ldots\ldots & = \frac{28}{10000} \\ & \ldots\ldots & = \cdot 0028. \end{array}$$

which consists of as many places of decimals as there were places of decimals both in the multiplier and multiplicand.

### Examples.

1. Multiply ·0725 by ·025.

$$\begin{array}{r} \cdot 0725 \\ \underline{\cdot 025} \\ 3625 \\ \underline{1450\phantom{0}} \\ \cdot 0018125 \end{array}$$

**Explanation.** In the preceding example the multiplicand has 4 places of decimals, and the multiplier 3, therefore the product must have 4+3 or 7 places of decimals, it was therefore necessary to add 2 ciphers to the left hand of the product 18125 to give the true product.

2. Multiply 3·75 by 2·8.
3. Multiply 14·376 by ·0049.
4. Multiply ·07345 by ·0435.
5. Multiply 47·1346 by ·0238.
6. Multiply 1547 by ·00463.

## DIVISION OF DECIMALS.

**Rule.** Divide as in whole numbers, and, from the right hand of the quotient, point off as many places for decimals as the dividend has places of decimals more than the divisor.

**Observations** (1). If the number of places of figures in the quotient is less than the required number of decimal places, then, as many *ciphers* as will supply the deficiency *must* be added to the *left hand* of the quotient, to give the true quotient.

(2). If the divisor has more places of decimals than the dividend, then, before the division is begun, ciphers *must* be added to the *right hand* of the dividend, till the number of decimal places in the dividend are, at least, *equal* to the number of decimal places in the divisor.

(3). If there should be a remainder after division, ciphers may be added to the dividend, and the operation carried on to any extent required.

The proof of this rule may be shewn, thus ;

Divide ·032 by ·04.

$$
\begin{aligned}
\text{Now}\quad & \cdot032=\tfrac{32}{1000}\\
\text{and}\quad & \cdot04=\tfrac{4}{100}\\
\text{therefore}\quad & \cdot032\div\cdot04=\tfrac{32}{1000}\div\tfrac{4}{100}\\
\ldots\ldots\ldots & =\frac{\overset{8}{32}}{\underset{10}{1000}}\times\frac{\overset{1}{100}}{\underset{1}{4}}\\
\ldots\ldots\ldots & =\tfrac{8}{10}\\
& =\cdot8
\end{aligned}
$$

which consists of as many places of decimals as the dividend has more than the divisor.

Again, since the number of decimal places in the dividend is equal to the number of decimal places in the divisor added to the number of decimal places in the quotient, ( for the dividend is equal to the quotient multiplied by the divisor,) the number of decimal places in the quotient will be equal to the number of decimal places in the divisor subtracted from the number of decimal places in the dividend.

## EXAMPLES.

1. Divide 3·17543 by ·09.

·09) 3·1754300

35·28255 &c. *Answer.*

EXPLANATION. In the preceding example since the divisor (·09) consists of a decimal with a cipher towards the left hand, the dividend is divided by the 9 in the divisor, and no notice is taken of the cipher until the decimal places in the quotient are pointed off. Again, since the dividend has 7 places of decimals, and the divisor 2, therefore the quotient will have 7—2, or 5 places of decimals.

2. Divide 17 by 47.

```
47) 17·0000000 (·3617021.
    14 1
    ----
     290
     282
     ---
       80
       47
      ---
      330
      329
      ---
        100
         94
        ---
          60
          47
          --
          13
```

3. Divide 674·3273 by 47·9.
4. Divide 13·49263 by ·004.
5. Divide 2 by ·7145.
6. Divide 17 by 823.
7. Divide ·36149 by 78·35.
8. Divide 83·27 by ·000496.
9. Divide 19·2638 by 47·315.
10. Divide 27 by ·008254.

## REDUCTION OF DECIMALS.

**Reduction of Decimals** may be divided into *three* cases.

**Case 1.** To reduce a vulgar fraction to a decimal *fraction of* equal value.

**Rule.** Reduce the fractions, when necessary, to simple fractions. Divide the numerator by the denominator, as in Division of Decimals.

### EXAMPLES.

1. Reduce $\frac{1}{7}$ to a decimal fraction.

$$7)\,\underline{1{\cdot}00000000}$$
$$\underline{\underline{{\cdot}14285714}}\ \text{\&c.}\ \textit{Answer.}$$

2. Reduce $27\frac{5}{12}$ to a mixed decimal.

$$12)\,\underline{5{\cdot}00000}$$
$$\underline{\underline{{\cdot}41666}}\ \text{\&c.}$$

$27\frac{5}{12}=27{\cdot}41666.$

3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ to decimal fractions.
4. Reduce $\frac{3}{17}$ to a decimal fraction.
5. Reduce $\frac{1}{3}$ of $\frac{4}{7}$ of $\frac{3}{8}$ to a decimal fraction.
6. Reduce $3\frac{7}{9}$ to a mixed decimal.
7. Reduce $3\frac{5}{7}$ of $\dfrac{4\frac{1}{8}}{3\frac{1}{3}}$ to a mixed decimal.
8. Reduce $\dfrac{4\frac{1}{7}}{6\frac{5}{9}}$ to a decimal fraction.

**Case 2.** To reduce quantities of different denominations to decimal fractions of a given denomination.

**Rule.** Reduce the lowest denomination to the next higher by dividing by as many of that lower denomination as make *one* of the next higher, and to the *left hand* of the quotient, which will be a decimal fraction, prefix the parts of that higher denomination, which reduce to the next higher, and so on, till a decimal fraction of the required denomination has been obtained.

This Rule is merely an application of the rule for Simple Reduction (Case 2,) and will be better understood from the following example.

### EXAMPLES.

1. Reduce 7*s.* 11¼*d.* to the decimal of a pound.

4) 1·000000 farthings
12) 11·250000 pence
20) 7·937500 shillings
·396875 pounds.

7*s.* 11¼*d.*=£ ·396875 *Answer.*

EXPLANATION. In the preceding example 1 farthing is divided by 4, because 1 farthing=¼ of a penny and the quotient is ·25 of a penny, to which the number of pence (11) is prefixed, and this sum 11·25 of a penny is divided by 12, because 1 penny=$\frac{1}{12}$ of a shilling, and therefore 11·25 of a penny=·9375 of a shilling, to which the number of shillings (7) is prefixed, and this sum 7·9375 of a shilling is divided by 20, because 1 shilling=$\frac{1}{20}$ of a pound, and therefore 7·9375 of a shilling=·396875 of a pound.

2. Reduce 2 oz. 7$\frac{3}{7}$ dwt. to the decimal of a pound (troy).

7) 3·00000000 dwts.
20) 7·42857142 dwts.
12) 2·37142857 ounces.
·19761904+pound troy.

*2 oz.* 7$\frac{3}{7}$ *dwt.*=·19761904+of a pound troy. *Answer.*

3. Reduce 4*s*. $7\frac{1}{4}$*d*. to the decimal of a pound.

4. Reduce 5*s*. $9\frac{3}{8}$*d*. to the decimal of a pound.

5. Reduce $8\frac{3}{4}$*d*. to the decimal of a pound.

6. Reduce 2 oz. 6 dwt. 14 gr. to the decimal of a pound (troy).

7. Reduce 3 dr. $2\frac{1}{5}$ sc. to the decimal of a pound (apothecaries').

8. Reduce 1 qr. 17 lb. $13\frac{1}{7}$ oz. to the decimal of a ton.

9. Reduce 2 qr. 1 nail to the decimal of an ell English.

10. Reduce 173 yds. 2 ft. 3 in. to the decimal of a mile.

11. Reduce 38 gal. $2\frac{1}{6}$ qt. to the decimal of a hogshead of wine.

12. Reduce 137 days 5 hrs. 16 min. to the decimal of a year.

13. Reduce 8*s*. $5\frac{1}{4}$*d*. to the decimal of a guinea.

14. What decimal part of a hundred-weight is 2 qrs. 15 lb. 7 oz.?

15. What decimal part of a mile is 19 poles 3 yds. 2 feet 4 inches?

CASE 3. To find the value of a decimal fraction in the known parts of the integer.

RULE. Multiply the decimal by as many of the next lower denomination as make one of the higher, and the *decimal places* in the product by as many of the next lower denomination as make *one* of that higher, and so on, as far as it can be done. The whole numbers in each product placed in order will be the value of the decimal fraction required.

This rule is merely an application of the rule for Simple Reduction (Case 1).

## EXAMPLES.

1. What is the value of ·73625 of a pound?

$$\begin{array}{r l} \cdot73625 & \\ 20 & \\ \hline 14\cdot72500 & \text{shillings} \\ 12 & \\ \hline 8\cdot70000 & \text{pence} \\ 4 & \\ \hline 2\cdot80000 & \text{farthings.} \\ \hline\hline \end{array}$$

£·73625=14*s*. 8½*d*. 8 or 14*s*. 8½.⅘*d*. *Answer.*

2. What is the value of ·31945 of a £?
3. What is the value of ·26373 of a guinea?
4. What is the value of ·4632 of a crown?
5. What is the value of ·0375 of a £?
6. What is the value of ·90762 of a lb. (troy)?
7. What is the value of ·57189 of a ton?
8. What is the value of ·17645 of a yard?
9. What is the value of ·647392 of a league?
10. What is the value of ·1375 of an acre?
11. What is the value of ·04793 of a hhd. of wine?
12. What is the value of ·306205 of a year?
13. Required the sum of ·13825 of a pound, 4·362 of a shilling, and 7·3 of a penny.
14. Required the sum of 19·0432 of a pound, 7·159 of a shilling, and 1·0473 of a penny.
15. What is the difference between 13·1735 of a *shilling,* and ·93875 of a pound?

16. Required the sum of 7·143 of a cwt., ·04715 of a quarter, and 8·8 of a pound (avoirdupois).

## RULE OF THREE IN DECIMALS.

**Rule.** State the question as in the common Rule of Three. Reduce the first and second terms to decimals of the highest denomination mentioned in either of them, and the third term to the decimal of the highest denomination mentioned in it. Multiply the second and third terms together, and divide their product by the first; the quotient will be in the same denomination that the third term was reduced to. Find the value of the decimal in the quotient, if any, by the former rule.

**Observation.** It will be found advantageous to state the question a second time, after the three terms have been reduced, as in the Rule of Three in Vulgar Fractions. Should more examples be required, the examples given in the Simple Rule of Three may be worked by Decimals.

### Examples.

1. If ¾ of a yard of satin cost 5*s.* 7½*d.*, what will 23½ yards cost at the same rate?

*yd.* *yds.* *s.* *d.*
As ¾ : 23½ :: 5 7½ : number required.

4) 3·00
·75 yard

2) 1·0
·5 yard

2) 1·00000
12) 7·50000
20) 5·62500
£·28125

```
     yd.     yd.        £
As   ·75  :  23·5  ::  ·28125  :  num. required.
                        23·5
                       ------
                       140625
                       84375
                      56250        £
                     --------
                 ·75) 6·609375 (8·8125
                      6 00          20
                     ---        -------
                        609     16·2500
                        600          12
                       ----     -------
                          93     3·0000
                          75     ======
                         ---
                          187
                          150
                          ---
                           375
                           375
                           ===
```

*Answer*. £8 16*s*. 3*d*.

2. What is the cost of 2 cwt. 1 qr. 14 lb. of sugar at £3·7514 per cwt.?

3. What is the value of a gold snuff-box weighing 4 oz. 7·145 dwt. at the rate of £4 17*s*. 6*d*. per ounce?

4. How much silk that is $\frac{7}{8}$ of a yard wide will line a cloak that is 1 yd. 3 qrs. long, and 9 qrs. 3 nails broad?

5. If a wedge of gold weighing 15$\frac{3}{8}$ pounds (troy) be worth £736·1735, what is the value of 1 dwt.?

6. Goliath, the Philistine, was 6$\frac{1}{4}$ cubits in height, what was his height in English feet, considering the cubit to have been 1 foot 7·168 inches?

7. The rental of a parish is £3759 11*s*.; the sum of £435 13*s*. 8*d*. is required for the support of the poor, what must a farmer pay towards the rate, who holds a farm of 315 acres at the yearly rent of £1 11*s*. 6*d*. *per* acre?

# PRACTICE.

**Practice** is a short method of performing questions in the Rule of Three, where the first term is an unit (1).

When a part is contained a certain number of times exactly in any number, that part is called *an aliquot part*, thus, ½ is an aliquot part of 1, for it is contained twice in 1; 3 is an aliquot part of 12, for it is contained 4 times in 12; &c., &c.

## TABLE OF ALIQUOT PARTS.

### *The aliquot parts of a Pound Sterling.*

| | | | | | |
|---|---|---|---|---|---|
| 10*s*. | is | ½ | 3*s*. 4*d*. | is | 1/6 |
| 6*s*. 8*d*. | is | ⅓ | 2*s*. 6*d*. | is | ⅛ |
| 5*s*. | is | ¼ | 2*s*. | is | 1/10 |
| 4*s*. | is | ⅕ | 1*s*. 8*d*. | is | 1/12 |

### *The aliquot parts of a Shilling.*

| | | | | | |
|---|---|---|---|---|---|
| 6*d*. | is | ½ | 2*d*. | is | 1/6 |
| 4*d*. | is | ⅓ | 1½*d*. | is | ⅛ |
| 3*d*. | is | ¼ | 1*d*. | is | 1/12 |

### *The aliquot parts of a Penny.*

| | | | | | |
|---|---|---|---|---|---|
| ½*d*. | is | ½ | ¼*d*. | is | ¼ |

### *The aliquot parts of a Hundred-weight.*

| | | | | | |
|---|---|---|---|---|---|
| 2 *qrs*. | is | ½ | 14 *lbs*. | is | ⅛ |
| 1 *qr*. | is | ¼ | 7 *lbs*. | is | 1/16 |

### *The aliquot parts of a Quarter.*

| | | | | | |
|---|---|---|---|---|---|
| 14 *lbs.* | is | ½ | 4 *lbs.* | is | 1/7 |
| 7 *lbs.* | is | ¼ | 3½ *lbs.* | is | ⅛ |

### *The aliquot parts of a Pound (avoirdupois).*

| | | | | | |
|---|---|---|---|---|---|
| 8 *oz.* | is | ½ | 2 *oz.* | is | ⅛ |
| 4 *oz.* | is | ¼ | 1 *oz.* | is | 1/16 |

*&c. &c. &c.*

For the convenience of the learner this rule is divided into *seven* cases.

Case 1. When the price is less than 1 penny.

Rule. Divide the given number by the aliquot part or parts of a penny, and the quotient or sum of the quotients will be in pence, which reduce to the highest name it admits of.

### Examples.

1. What is the price of 73695 at ¾*d.*?

½*d.* is ½ 73695
¼*d.* is ½ 36847 ½
18423 ¾
12) 55271 ¼
2,0) 460,5 11¼
£230 5 11¼

*Answer.* £230 5*s.* 11¼*d.*

Explanation. 73695 at 1*d.* will cost 73695*d.*; 73695 at ½*d.* will cost the half of 73695*d.* (36847½*d.*), *because* ½*d.* is the half of 1*d.*; 73695 at ¼*d.* will cost the half of 36847½*d.* (18423¾*d.*), because ¼*d.* is the half of ½*d.*; and 73695 at ¾*d.* will cost 55271¼*d.* which reduced is equal to £230 5*s.* 11¼*d.*

2. What is the price of 641753 at ¼*d.*?
3. What is the price of 43615 at ½*d.*?
4. What is the price of 27637 at ¾*d.*?

CASE 2. When the price is less than 1 shilling.

RULE. Divide the given number by the aliquot part or parts of a shilling, the quotient or sum of the quotients will be in shillings, which reduce, if necessary, to pounds.

## EXAMPLES.

1. What is the price of 4173 at 3*d.*?

| | | | |
|---|---|---|---|
| 3*d.* is ¼ | 4173 | | |
| 2,0) | 104,3 | | 3 |
| | £52 | 3 | 3 |

*Answer.* £52 3*s.* 3*d.*

EXPLANATION. 4173 at 1*s.* will cost 4173 shillings; 4173 at 3*d.* will cost ¼ of 4173*s.* (1043*s.* 3*d.*), because 3*d.* is ¼ of 1 shilling.

2. What is the value of 3793 at 7¼*d.*?

| | | | |
|---|---|---|---|
| 6*d.* is ½ | 3793 | | |
| 1*d.* is ⅙ | 1896 | | 6 |
| ¼*d.* is ¼ | 316 | | 1 |
| | 79 | | 0¼ |
| 2,0) | 229,1 | | 7¼ |
| | £114 | 11 | 7¼ |

*Answer.* £114 11*s.* 7¼*d.*

**Explanation.** Here 6*d.* is ½ of 1*s.*; 1*d.* is ⅙ of 6*d.*; and ¼*d.* is ¼ of 1*d.*

3. What is the price of 6735 pairs of gloves at 11¾*d.* per pair.

| | | *s.* | *d.* | | *s.* | | £ | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|
| 6735 | at | 1 | 0 | = | 6735 | = | 336 | 15 | 0 |
| | | | | | *farthings* | | | | |
| 6735 | at | 0 | 0¼ | = | 6735 | = | 7 | 0 | 3¾ |
| 6735 | at | 0 | 11¾ | | | = | £329 | 14 | 8¼ |

**Explanation.** Here the value of 6735 at ¼*d.* is subtracted from the value of 6735 at 1*s.*, the difference will be the value of 6735 at 11¾*d.*

4. What is the price of 3759 at 1½*d.*?
5. What is the price of 6847 at 2*d.*?
6. What is the price of 17963 at 2½*d.*?
7. What is the price of 7489 at 3¾*d.*?
8. What is the price of 51763 at 4*d.*?
9. What is the price of 6759 at 5½*d.*?
10. What is the price of 73652 at 6¾*d.*?
11. What is the price of 19231 at 7¾*d.*?
12. What is the price of 62753 at 8¼*d.*?
13. What is the price of 27615 at 9¾*d.*?
14. What is the price of 6148 at 10½*d.*?
15. What is the price of 27363 at 11½*d.*?
16. What is the price of 43025 at 11¾*d.*?

**Case 3.** When the price is any number of shillings.

**Rule.** If the price is an aliquot part of a pound, divide the given number by that aliquot part, and the quotient will be the answer in pounds.

If the price is not an aliquot part of a pound, multiply the given number by the price, and reduce the product into pounds.

### EXAMPLES.

1. What is the price of 7365 at 4*s*.?

$$\begin{array}{rr} 4s.\ \text{is}\ \tfrac{1}{5} & \underline{7365} \\ & \underline{\underline{£1473}} \end{array}$$

*Answer.* £1473.

EXPLANATION. Here 4*s*. being ⅕ of a pound, the price of 7365 at 4*s*. will be the ⅕ of £7365 (£1473).

2. What is the price of 6147 yards of silk at 7*s*. per yard?

$$\begin{array}{rr} & 6147 \\ & \underline{7} \\ 2{,}0) & \underline{4302{,}9} \\ & \underline{\underline{£2151\quad 9}} \end{array}$$

*Answer.* £2151 9*s*.

3. What is the value of 4273 at 3*s*.?
4. What is the value of 8243 at 4*s*.?
5. What is the value of 6179 at 5*s*.?
6. What is the value of 7385 at 8*s*.?
7. What is the value of 5032 at 9*s*.?
8. What is the value of 1763 at 10*s*.?
9. What is the value of 2915 at 13*s*.?
10. What is the value of 4703 at 15*s*.?
11. What is the value of 3649 at 17*s*.?
12. What is the value of 2327 at 19*s*.?

CASE 4. When the price is any number of shillings and pence, but less than 1 pound.

RULE. Divide the given number by the aliquot part or parts of a pound, and the quotient, or sum of the quotients, will be the answer in pounds.

## EXAMPLES.

1. What is the value of 3753 pairs of stockings at 1*s*. 8*d*. per pair?

| | | | |
|---|---|---|---|
| 1*s*. 8*d*. is $\frac{1}{12}$ | 3753 | | |
| | £312 | 15 | |

*Answer.* £312 15*s*.

EXPLANATION. Here 1*s*. 8*d*. being $\frac{1}{12}$ of a pound, the price of 3753 at 1*s*. 8*d*. will be the $\frac{1}{12}$ of £3753 (£312 15*s*.).

2. If I buy cloth at 17*s*. 10½*d*. per yard, what must I pay for 973 yards?

| | | | |
|---|---|---|---|
| 10*s*. 0*d*. is ½ | 973 | | |
| 5*s*. 0*d*. is ½ | 486 | 10 | |
| 2*s*. 6*d*. is ½ | 243 | 5 | |
| 3*d*. is $\frac{1}{10}$ | 121 | 12 | 6 |
| 1½*d*. is ½ | 12 | 3 | 3 |
| | 6 | 1 | 7½ |
| | £869 | 12 | 4½ |

*Answer.* £869 12*s*. 4½*d*.

EXPLANATION. Here 10*s*. is ½ of £1; 5*s*. is ½ of 10*s*.; 2*s*. 6*d*. is ½ of 5*s*.; 3*d*. is $\frac{1}{10}$ of 2*s*. 6*d*.; and 1½*d*. is ½ of 3*d*.

3. Find the value of 736 lambs at 19*s*. 11¾*d*. each.

| | | £ | *s*. | *d*. | | £ | *s*. | *d*. |
|---|---|---|---|---|---|---|---|---|
| 736 | at | 1 | 0 | 0 | = | 736 | 0 | 0 |
| 736 | at | 0 | 0 | 0¼ | = | 0 | 15 | 4 |
| 736 | at | 0 | 19 | 11¾ | = | 735 | 4 | 8 |

*Answer.* £735 4*s*. 8*d*.

EXPLANATION. Here the price of 736 at ¼*d.* is subtracted from the price of 736 at £1; the difference is the price of 736 at 19*s.* 11¾*d.*

| | | | | *s.* | *d.* |
|---|---|---|---|---|---|
| 4. | What is the value of | 6943 | at | 2 | 6? |
| 5. | What is the value of | 7849 | at | 3 | 4? |
| 6. | What is the value of | 18261 | at | 6 | 8? |
| 7. | What is the value of | 8067 | at | 3 | 7½? |
| 8. | What is the value of | 6405 | at | 4 | 8¾? |
| 9. | What is the value of | 87362 | at | 5 | 11¾? |
| 10. | What is the value of | 2043 | at | 6 | 3½? |
| 11. | What is the value of | 5707 | at | 7 | 5¼? |
| 12. | What is the value of | 19324 | at | 11 | 2½? |
| 13. | What is the value of | 1373 | at | 14 | 9½? |
| 14. | What is the value of | 8061 | at | 17 | 10¾? |
| 15. | What is the value of | 2805 | at | 18 | 7½? |
| 16. | What is the value of | 6107 | at | 19 | 1¾? |
| 17. | What is the value of | 15329 | at | 19 | 11¼? |
| 18. | What is the value of | 7048 | at | 19 | 11¾? |

CASE 5. When the price is any number of pounds, shillings, and pence.

RULE. Multiply the given number by the number of pounds in the price, and take the aliquot parts for the shillings and pence as in the last case.

### EXAMPLES.

1. What is the price of 739 hundred-weight of sugar at £5 7*s.* 9½*d.* per hundred-weight?

| | | | |
|---|---|---|---|
| | 739 | | |
| | 5 | | |
| | 3695 | | |
| 5*s.* 0*d.* is ¼ | 184 | 15 | 0 |
| 2*s.* 6*d.* is ½ | 92 | 7 | 6 |
| 3*d.* is $\frac{1}{10}$ | 9 | 4 | 9 |
| ½*d.* is $\frac{1}{6}$ | 1 | 10 | 9½ |
| | £3982 | 18 | 0½ |

*Answer.* £3982 18*s.* 0½*d.*

| | | £ | s. | d. |
|---|---|---|---|---|
| 2. | What is the price of 8173 at | 1 | 17 | 6 ? |
| 3. | What is the price of 2690 at | 2 | 13 | 7¼ ? |
| 4. | What is the price of 7137 at | 3 | 14 | 8¾ ? |
| 5. | What is the price of 6049 at | 7 | 11 | 4½ ? |
| 6. | What is the price of 3715 at | 6 | 15 | 9¾ ? |
| 7. | What is the price of 8073 at | 11 | 16 | 2¼ ? |
| 8. | What is the price of 2614 at | 17 | 7 | 7½ ? |
| 9. | What is the price of 1927 at | 23 | 18 | 1 ? |
| 10. | What is the price of 8423 at | 47 | 16 | 5½ ? |
| 11. | What is the price of 2805 at | 19 | 19 | 11½ ? |
| 12. | What is the price of 4937 at | 24 | 19 | 11¾ ? |
| 13. | What is the price of 1832 at | 9 | 16 | 8 ? |
| 14. | What is the price of 7263 at | 13 | 7 | 10½ ? |
| 15. | What is the price of 6488 at | 7 | 13 | 4 ? |
| 16. | What is the price of 7387 at | 21 | 16 | 7¼ ? |

CASE 6. When the quantity of which the value is required is a mixed number.

RULE. Find the value of the whole number by the preceding cases, to which add the value of the fractional part.

### EXAMPLES.

1. What is the price of 273¾ at 3*s*. 7*d*.

| | | | |
|---|---|---|---|
| 2*s*. 6*d*. is ⅛ | 273 | | |
| 1*s*. 0*d*. is $\frac{1}{20}$ | 34 | 2 | 6 |
| 1*d*. is $\frac{1}{12}$ | 13 | 13 | 0 |
| | 1 | 2 | 9 |
| for ½ | 0 | 1 | 9½ |
| for ¼ | 0 | 0 | 10¾ |
| | £49 | 0 | 11¼ |

Answer. £49 0*s*. 11¼*d*.

2. What is the price of $714\frac{5}{11}$ at 2*s.* 6*d.*?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | *s.* | *d.* |
| | | | | | | 2 | 6 |
| 2*s.* 6*d.* is $\frac{1}{8}$ | 714 | | | | | | 5 |
| | 89 | 5 | | | 11) | 12 | 6 |
| for $\frac{5}{11}$ | 0 | 1 | $1\frac{1}{2}.\frac{6}{11}$ | | | 1 | $1\frac{1}{2}.\frac{6}{11}$ |
| | £89 | 6 | $1\frac{1}{2}.\frac{6}{11}$ | | | | |

*Answer.* £89 6*s.* $1\frac{1}{2}.\frac{6}{11}$*d.*

| | | £ | *s.* | *d.* |
|---|---|---|---|---|
| 3. | What is the price of $623\frac{1}{4}$ at | 0 | 4 | 8? |
| 4. | What is the price of $714\frac{3}{4}$ at | 0 | 6 | 8? |
| 5. | What is the price of $835\frac{1}{4}$ at | 0 | 17 | 6? |
| 6. | What is the price of $2714\frac{1}{2}$ at | 0 | 2 | $7\frac{1}{2}$? |
| 7. | What is the price of $1832\frac{3}{4}$ at | 0 | 3 | 4? |
| 8. | What is the price of $1936\frac{3}{4}$ at | 0 | 19 | $11\frac{1}{2}$? |
| 9. | What is the price of $6273\frac{1}{6}$ at | 1 | 13 | 4? |
| 10. | What is the price of $1705\frac{3}{8}$ at | 0 | 8 | $7\frac{1}{4}$? |
| 11. | What is the price of $3176\frac{5}{9}$ at | 2 | 15 | $5\frac{3}{4}$? |
| 12. | What is the price of $8201\frac{2}{7}$ at | 6 | 4 | 9? |
| 13. | What is the price of $1759\frac{6}{11}$ at | 13 | 19 | $3\frac{1}{2}$? |
| 14. | What is the price of $6103\frac{1}{12}$ at | 8 | 13 | $8\frac{3}{4}$? |

CASE 7. When the quantity of which the value is required is of several denominations.

RULE. Multiply the given price by the highest denomination as in Compound Multiplication, and take parts of the given price for the lower denominations.

## EXAMPLES.

1. What is the cost of 7 cwt. 2 qrs. 17 lbs. of sugar at £4 13*s.* 9*d.* per cwt.?

|  | £ | s. | d. |
|---|---|---|---|
|  | 4 | 13 | 9 |
|  |  |  | 7 |
|  | 32 | 16 | 3 |
| 2 qrs. is ½ | 2 | 6 | 10½ |
| 14 lbs. is ¼ | 0 | 11 | 8½ |
| 2 lbs. is 1/7 | 0 | 1 | 8 |
| 1 lb. is ½ | 0 | 0 | 10 |
|  | £35 | 17 | 4 |

*Answer.* £35 17*s.* 4*d.*

2. What is the value of 3 cwt. 2 qrs. 14 lbs. of sugar at £3 2*s.* 6*d.* per cwt.?

3. What is the value of 7 cwt. 3 qrs. 18 lbs. of soap at £3 14*s.* 9*d.* per cwt.?

4. What is the value of 17 oz. 13 dwt. of gold at £3 19*s.* 10½*d.* per ounce?

5. What is the value of 169 oz. 15 dwt. of silver at 5*s.* 9¼*d.* per ounce?

6. What is the value of 12 hhds. 21 galls. of sherry at £41 12*s.* 4*d.* per hogshead?

7. What is the value of 136 tons 14 cwt. 3 qrs. of iron at £13 7*s.* 8*d.* per ton?

8. What is the value of 89 yds. 3 qrs. 2 n. of silk at 5*s.* 4¾*d.* per yard?

9. What is the value of 173 E.e. 1 qr. 3 n. of satin at 13*s.* 5¼*d.* per ell?

10. What is the value of 67 yds. 3 qrs. 3 n. of lace at 18*s.* 3½*d.* per yard?

## INTEREST.

INTEREST is a certain sum paid, by the borrower to the lender, for the use of any sum of money for any fixed time.

Interest is generally calculated at so much per centum, per annum. (*Per centum*, for £100; *per annum*, for 1 year.)

The following terms are made use of in calculating interest.

1. *Principal* is the money originally lent.

2. *Rate per cent., or per cent.* is the sum agreed to be paid for the use of £100 for *one* year.

3. *Amount* is the interest added to the principal. Thus. 5 per cent., per annum, means that £5 is to be paid, by the borrower to the lender, for the use of £100 for one year, so that the lender of £100 will have due to him, at the end of 1 year, £105, which sum is called the *amount*.

Interest is of *two* kinds Simple and Compound.

*Simple Interest* is where the interest is paid only on the principal.

*Compound Interest* is where the interest at a certain fixed time, when due, is added to the principal, and that amount forms a new principal on which the interest for the next period is calculated, which interest is again added to that amount to form the principal for the next period, and so on.

## SIMPLE INTEREST.

Rule. Find the interest of the given sum for one year by the Rule of Three, or what is the same thing, multiply the principal by the rate per cent. and divide the product by 100, the quotient will be the interest for one year.

The interest for any number of years may then be found by multiplying the interest for one year by the number of years, and for any parts of a year by Practice or the Rule of Three.

## EXAMPLES.

1. What is the interest of £347 16*s*. for 2 years, 7 months, and 15 days at 5 per cent. per annum?

| | £ | | £ | *s*. | | £ | | |
|---|---|---|---|---|---|---|---|---|
| As | 100 | : | 347 | 16 | :: | 5 | : | int. for 1 yr. |
| | | | | 5 | | | | |

100) 17,39
20
7,80
12
9,60
4
2,40

| | | £ | *s*. | *d*. | | |
|---|---|---|---|---|---|---|
| | | 17 | 7 | 9½ | ·4 | interest for 1 year |
| | | | | 2 | | |
| | | 34 | 15 | 7 | ·8 | interest for 2 years |
| 6 months | is ½ | 8 | 13 | 10¾ | ·2 | interest for 6 mths. |
| 1 month | is ⅙ | 1 | 8 | 11¾ | ·2 | interest for 1 mth. |
| 15 days | is ½ | 0 | 14 | 5¾ | ·6 | interest for 15 days. |
| | | £45 | 12 | 11½ | ·8 | |

*Answer*. £45 12*s*. 11½*d*. ·8.

EXPLANATION. Here, as £100 : £347 16*s*. :: £5 (the interest of £100 for 1 year) : the interest of £347 16*s*. for 1 year (£17 7*s*. 9½*d*. ·4). The interest for 1 year is then multiplied by 2 to find the interest *for two years*, and parts are taken for the 7 months and

15 days. Account has been taken of the fractional parts, which is not necessary for practical purposes.

The same example by Decimals.

£ £ £
As 100 : 347·8 :: 5 : interest for 1 year
5
100) 1739
days 17·39 interest for 1 year
30) 15·000 2·625
12) 7·500 8695
3478
·625 10434
3478
45·64875
20
12·97500
12
11·700
4
2·8

*Answer.* £45 12*s.* 11½*d.* ·8.

2. What is the interest of £364 17*s.* for 67 days at 4 per cent. per annum?

£ £ *s.* £
As 100 : 364 17 :: 4 : int. for 1 year

days days
then 365 : 67 :: int. for 1 yr. : int. for 67 days.

3. Find the interest of £496 13*s.* 4*d.* for 3 years at 5 per cent. per annum.

4. Find the interest of £327 17*s.* 11*d.* for 4½ years at 4½ per cent. per annum.

5. Find the interest of £526 13*s.* for 2 years and 3 months at 3½ per cent. per annum.

6. Find the interest of £173 19*s.* 8½*d.* for 3 years and 7 months at 4½ per cent. per annum.

7. Find the interest of £235 14*s.* 9*d.* for 4 years, 5 months, and 17 days at 3¾ per cent. per annum.

8. Find the interest of £368 9*s.* 10½*d.* for 7 years, and 189 days at 4¾ per cent. per annum.

9. Required the interest of £738 19*s.* 8*d.* for 5 years and 237 days at 3½ per cent. per annum.

10. A gentleman, by his will, left his son, who, at his father's decease, was 22 years and 149 days old, the sum of £17400 to be paid him with interest upon the same at 3½ per cent. per annum when he attained his 25th year. What would he have to receive?

11. A gentleman dies leaving one son, aged 14¼ years, his property after payment of all debts amounts to £15847 15*s.* The executors put the money out on mortgage, and get 4½ per cent. per annum, the interest according to the provision of the will was to be disposed of in the following manner: £250 annually for the young gentleman's education, &c., and the remainder to form a building fund against he came of age, the executors having the interest of that fund for their trouble. What will he have to receive when he comes of age?

12. A manufacturer wishing to increase his capital, in order to carry on a particular business requiring a large outlay, borrows of his banker, on the 20th of May, 1837, the sum of £5850 at 4¾ per cent., the interest to be paid half-yearly. On the 6th of September, 1840, the banker dies suddenly, three weeks after, his executors close the account. What will be the amount due to the estate, supposing the interest to have been regularly paid?

# COMPOUND INTEREST.

**RULE.** Find the amount of the given principal for the first period as in Simple Interest. This amount is the principal for the second period. Find the amount of this new principal, which will be the principal for the third period, and so on through all the periods. Subtract the given principal from the last amount, and the remainder will be the compound interest required.

## EXAMPLES.

1. What is the compound interest of £735 for 3 years at 4 per cent. per annum?

|  | £ |  | £ |  | £ |  |  |
|---|---|---|---|---|---|---|---|
| As | 100 | : | 735 | :: | 4 | : | 1st year's interest |
|  |  |  | 4 |  |  |  |  |

100) 29,40
20
8,00

| £ | s. |  |
|---|---|---|
| 735 | 0 | 1st year's principal |
| 29 | 8 | 1st year's interest |
| 764 | 8 | 2nd year's principal |

|  | £ |  | £ | s. |  | £ |  |  |
|---|---|---|---|---|---|---|---|---|
| As | 100 | : | 764 | 8 | :: | 4 | : | 2nd year's interest |
|  |  |  |  | 4 |  |  |  |  |

100) 30,57 12
20
11,52
12
6,24

| £ | s. | d. | |
|---|---|---|---|
| 764 | 8 | 0 | 2nd year's principal |
| 30 | 11 | 6 | 2nd year's interest |
| 794 | 19 | 6 | 3rd year's principal |

| | £ | | £ | s. | d. | | £ | | |
|---|---|---|---|---|---|---|---|---|---|
| As | 100 | : | 794 | 19 | 6 | : | 4 | : | 3rd year's int. |
| | | | | | 4 | | | | |
| | 100) | | 31,79 | 18 | 0 | | | | |
| | | | | 20 | | | | | |
| | | | | 15,98 | | | | | |
| | | | | 12 | | | | | |
| | | | | 11,76 | | | | | |
| | | | | 4 | | | | | |
| | | | | 3,04 | | | | | |

| £ | s. | d. | |
|---|---|---|---|
| 794 | 19 | 6 | 3rd year's principal |
| 31 | 15 | 11¾ | 3rd year's interest |
| 826 | 15 | 5¾ | amount for 3 years |
| 735 | 0 | 0 | given principal |
| £91 | 15 | 5¾ | compound interest. |

*Answer.* £91 15*s.* 5¾*d.*

OBSERVATION. In the preceding example, no notice has been taken of the fractional parts of the farthings, they not being necessary for practical purposes.

The most correct and simple method of working an example in Compound Interest is as follows.

Taking the same example.

£ £ £
As 100 : 735 :: 4 : 1st year's interest
4
100) 2940
29·40 1st year's interest
735 1st year's principal
£ £
As 100 : 764·4 :: 4 : 2nd years' interest
4
100) 3057·6
30·576 2nd years' interest
764·4 2nd years' principal
£ £
As 100 : 794·976 :: 4 : 3rd years' interest
4
100) 3179·904
31·79904 3rd years' interest
794·976 3rd years' principal
826·77504 amount
735 given principal
91·77504 compound interest
20
15·50080
12
6·0096

*Answer.* £91 15*s.* 6*d.*

2. What is the amount of £337 13*s.* for 4 years at 5 per cent. per annum compound interest?

3. What is the compound interest of £745 for 3 years at 4½ per cent.?

4. What is the amount of £430 for 3 years at 3½ per cent.?

## COMMISSION. BROKERAGE. INSURANCE.

COMMISSION is an allowance of so much per cent. paid by merchants to their agents in foreign countries, or at a distance, for buying and selling goods. The rate per cent. varies according to the custom of the country where the agent resides. These agents are frequently called *Factors*, as Coal-Factors, Corn-Factors, &c., &c.

BROKERAGE is a similar allowance made to agents at home for buying and selling goods. These agents are called *Brokers*, and each generally buys or sells but one description of goods, as, wine, tea, sugar, &c.; hence the terms Wine-Broker, Tea-Broker, Sugar-Broker, &c.

INSURANCE is so much per cent. paid to companies or individuals, who engage for that per-centage to make good losses by fire, storms at sea, &c.

Persons who take insurances on ships, freight, &c., are called *Underwriters*, from their writing their names, the amount of their risk, &c., underneath the terms of the policy. Where the sum insured is large, several parties divide the risk. The sum paid is called the *premium*, and the paper containing the contract, the *policy*.

Examples in these rules are worked like those in Simple Interest.

### EXAMPLES.

1. What commission must I pay my factor for disposing of coals to the amount of £1753 at 2¼ per cent.?

$$\begin{array}{lrcrcrcl}
 & \pounds & & \pounds & & \pounds & & \\
\text{As} & 100 & : & 1753 & :: & 2\tfrac{1}{4} & : & \text{commission required.}
\end{array}$$

$$\begin{array}{rll}
 & \phantom{0}\phantom{0}2\tfrac{1}{4} & \\ \hline
 & 3506 & \\
 & \phantom{0}438 & 5 \\ \hline
100) & 39{,}44 & 5 \\
 & \phantom{0}\phantom{0}20 & \\ \hline
 & 8{,}85 & \\
 & \phantom{0}\phantom{0}12 & \\ \hline
 & 10{,}20 & \\ \hline\hline
\end{array}$$

*Answer*. £39 8*s*. 10⅕*d*.

2. What is the brokerage of £759 at ⅝ per cent?

$$\begin{array}{lrcrcrcl}
 & \pounds & & \pounds & & \pounds & & \\
\text{As} & 100 & : & 759 & :: & \tfrac{5}{8} & : & \text{brokerage required.}
\end{array}$$

$$\tfrac{759}{1}\times\tfrac{5}{8}\div\tfrac{100}{1}=\tfrac{759}{1}\times\tfrac{5}{8}\times\tfrac{1}{100}=\tfrac{3795}{800}=\pounds 4\ 14s.\ 10\tfrac{1}{2}d.$$

$$\begin{array}{lrcrcrcl}
 & \pounds & & \pounds & & \pounds & & \\
\text{or As} & 100 & : & 759 & :: & \tfrac{5}{8} & : & \text{brokerage required.}
\end{array}$$

$$\begin{array}{lrrrr}
 & & \tfrac{5}{8} & & \\ \hline
\tfrac{4}{8}\text{ is }\tfrac{1}{2} & & 379 & 10 & \\
\tfrac{1}{8}\text{ is }\tfrac{1}{4} & & 94 & 17 & 6 \\ \hline
 & 100) & 4{,}74 & 7 & 6 \\
 & & 20 & & \\ \hline
 & & 14{,}87 & & \\
 & & 12 & & \\ \hline
 & & 10{,}50 & & \\
 & & 4 & & \\ \hline
 & & 2{,}00 & & \\ \hline\hline
\end{array}$$

*Answer*. £4 14*s*. 10½*d*.

3. What is the insurance of £1500 at 1⅛ per cent.

$$\begin{array}{llllllll}
 & \pounds & & & \pounds\ s.\ d. & & \pounds\ s.\ d. \\
\text{Ins. of} & 100 & \text{at } 1\tfrac{1}{8} & \text{per cent} = & 1\ 17\ 6 & & \\
\text{Ins. of} & 1500 & \text{at } 1\tfrac{1}{8} & \text{per cent} = & 1\ 17\ 6 & \times 15 = & 28\ 2\ 6
\end{array}$$

4. What is the commission upon £2715 17*s.* at 2¼ per cent.?

5. What is the brokerage upon £483 15*s.* 8*d.* at ⅞ per cent.?

6. What is the insurance upon a ship and cargo, worth £18349 17*s.* at 7¾ per cent, stamp duty 2*s.* 6*d.* per cent.?

7. If I employ a broker to sell goods which fetch £843 8*s.* 6*d.*; what will be his allowance at 1⅜ per cent.?

8. What is the insurance of £1500 at 17⅝ per cent.?

9. If I employ a broker to sell goods to the amount of £1643 16*s.* 8*d.*; what is his allowance at 5*s.* 9*d.* per cent.?

10. A merchant possesses a cargo on board a ship homeward bound, to the value of £2870, which ship is much behind her time, and a violent storm is known to have raged in the latitude in which she is supposed to have been upon a certain day, in consequence of which the underwriters refuse to insure the cargo under 35 per cent. What will be the net value of the cargo, supposing the ship comes safe home, the stamp duty of 2*s.* 6*d.* per cent. being added to the insurance?

## BUYING AND SELLING OF STOCK.

STOCK; this term is used to signify the trading capital of several public companies, but more commonly the amount of several debts due from the Government to the Nation.

The stocks of public companies, called *capital*, are sums raised, in shares of a certain value, for the carrying on of some particular trade, requiring a very large outlay, and this capital, being limited to a certain sum by the Act of Parliament incorporating the company, it follows, that when this capital is raised, no more shares can be bought of the company. These shares may however be sold by the parties holding them, such sales are called *transfers*, the securities, or *papers acknowledging* the debt, being *transferred* from the one party to the other, such transfer being duly noted in the books of the company called *transfer books.* The prices of these shares rise and fall according to the profits, or estimated profits of the company.

When the shares are sold for more than their original cost, they are said to be at a *premium;* when for less, at a *discount;* and when for the same sum, at *par* (equality).

The interest paid on these shares is called a *dividend,* from the disposable profits being *divided* among the shareholders.

Government Securities, or, The Stocks, were originally large sums advanced to Government, on the security of the public taxes, to defray the expences of wars, &c., and commenced about the time of the Revolution in 1688. These loans, raised from time to time, have now risen to an enormous permanent debt, called *the National Debt,* the interest upon which is paid *half yearly* from the produce of the taxes. These half yearly payments are called *dividends,* and are paid at the Bank of England, the Bank having in return certain privileges.

Persons holding stock cannot demand payment of their portion of the debt in specie (gold and silver coin), but have the power of selling their stock, which sale takes place in a building called the *Stock Exchange,* by means of a person called a *Stock Broker,* who charges £⅛ per cent. for his trouble; the amount of stock is not altered by this sale, the particular amount of stock thus sold being merely transferred, in the Bank books, from the name of the party selling, to that of the party buying the stock.

The *transfer books* of any stock are shut about a month before the dividends on that stock become due, they remain shut about six weeks; the word *shut* means that no stock during that time can be transferred.

Stock may be transferred from one party to another without being put up for sale, thus, bequests and legacies of stock may be paid in stock by the executors of any person deceased, the stock being merely transferred from the name of the party deceased to that of the legatee.

The principal government securities are as follows; *The 3 per cent. Reduced Annuities* amounting to about 125 millions of stock, called *Reduced* from their having originally borne a higher rate of interest, which interest, having on various occasions been reduced, is now 3 per cent.

*The 3 per cent. Consolidated Annuities* amounting to about 363 millions of stock, called consolidated, or technically *consols*, from their being formed by the *consolidation* of several stocks. The amount of this stock on the 5th of January, 1837, was £357,166,327 4*s.* 11*d.*, which amount was increased in the year 1811 by the sum of £5,600,000.

*The* 3½ *per cent. Annuities,* 1818, amounting to about 11 millions.

*The* 3½ *per cent. Reduced Annuities,* amounting to about 71 millions. The amount of this stock on the 5th of January, 1837, was £66,273,320 15*s.* 3*d.*, which amount has since been increased by the sum of £5,000,000 to pay the claimants under the Slave Compensation Act.

*The New* 3½ *per cent. Annuities* amounting to about 146 millions.

Besides these *Permanent Annuities* there are three *Terminable Annuities*, one is called the *Long Annuity*, another the *April and October Annuity*, and the third the *January and July Annuity :* it is to the two last Annuities that the word *Terminable* is generally applied in "the market."

*The Long Annuities, January 5th,* 1860, these, as they will *terminate* on that day, become yearly of less value, being only worth so many years' purchase, the interest is 6 per cent. The amount of the *Long Annuities* on the 5th of January, 1837, was £1,294,586 6*s.* 10*d.*

The *April and October Annuity*, and the *January and July Annuity* are Annuities for various terms, *granted* at different times.

From a return laid before Parliament in the session of 1833 it appeared that the number of public creditors was 279,751, but as many of the accounts were in joint names ( trustees to individuals, public companies, &c.), the precise number of persons interested in the public funds could not be easily ascertained.

| | | | |
|---|---|---|---|
| The half-yearly dividend of | 87,176 | was under | £5 |
| | 44,648 | | £10 |
| | 98,305 | | £50 |
| | 25,641 | | £100 |
| | 14,701 | | £200 |
| | 4,495 | | £300 |
| | 2,827 | | £500 |
| | 1,367 | | £1000 |
| | 591 | was over | £1000 |

Of the 591 dividends which exceeded £1000, 210 were the dividends due to Public Companies, or to joint accounts.

The Stocks of the principal trading companies are as follows.

*India Stock* amounting to 6 millions, which is the capital of the East India Company, the dividend is 10½ per cent.

*Bank Stock* amounting to about 15 millions, which is the trading capital of the Bank of England, and with which they carry on the banking business, buy bullion ( uncoined gold and silver), &c., the dividend is now 7 per cent. Of this capital £11,015,100 is lent to Government at 3 per cent.

*South Sea Stock and Annuities* amounting to about 10 millions, the greater part of which is lent to Government, for which the company receive 3 per cent.; but from profits derived from other sources, the proprietors receive a dividend of 3½ per cent.

The prices of stock are constantly varying according to the prosperity of the country, the state of our foreign relations, the confidence reposed in the Ministers of the Crown, &c., &c.

A list of these prices appears daily in the public prints, and at shorter intervals at the offices of the principal Stock Brokers.

The following will serve as an example of these lists.

Bank Stock, 169½, 168½, 169½.
3 per cent. Red., 90⅛, ¼, 90.
3 per cent. Consols, 89⅞, ¾.
3½ per cent. Red., 99⅛, ¼.
New 3½ per cent. 98⅝, ½.
India Stock, 247½, 246½.
Ditto Bonds 1.4 pm.
Consols for account, 89⅝, ½.
Exchequer Bills, 2¼*d.*, 16*s.*, 18*s.* pm.

From this list we learn that £100 Bank Stock, during the day on which the list was made, could be bought for £169½ and sold for £168½, as in Bank and India Stocks the "Jobbers" usually make a "1 per cent. price," that is, if a Broker comes into "the market" wishing to buy Bank Stock, the "Jobbers" would make the price £169½, but if wishing to sell, then £168½: this would be called "a steady market." If in the course of the day Bank Stock was quoted at 170½, it would then be said to have risen 1 per cent. The prices being quoted 169½, 168½, 169½ imply that two purchases were made to one sale.

From the above list we also learn that £100 stock in the 3 per cent. Reduced sold at the beginning of the day for £90 2*s.* 6*d.*, and rose to £90 5*s.*, and again fell at the close of the day to £90, a fluctuation of ⅛ per cent.—that £100 stock in the 3 per cent. Consols sold at the beginning of the day for 89⅞ and left off at 89¾, a fall of ⅛ per cent.—and that £100 stock in the new 3½ per cent. sold at the beginning of the day for 98⅝ and left off at 98½, a fall of ⅛ per cent.

The dividends of the 3 per cent. Red. and the 3½ per cent Red. being due on the 5th of April and the 10th of October, and those of the 3 per cent. consols, and the new 3½ per cent., on the 5th of January and the 5th of July, cause the difference in price between the 3 per cent Red., and the 3 per cent. consols, &c., &c.

*Consols for the account* are sales of stock between speculators on the Stock Exchange, called *Stock Jobbers,* who buy and sell stock on one day, the said stock *supposed* to be transferred on some future day, they taking the chances of what may be the price of that stock, on the day on which it is *supposed* to be transferred. These transactions are called *Time Bargains*, and are frequently nothing more nor less than gambling transactions, the difference only being paid. Thus if A sells to B £10000 in any stock for £98 per cent. on a Monday for a subsequent Thursday, he is supposed to let B, on that Thursday have that amount of stock at the rate of £98 per cent., let the price of that particular stock on the Thursday be what it may; so that, if the particular stock in which this £10000 is *supposed* to be should rise on the Thursday to 99 per cent., or 1 per cent. higher than it was on the Monday, A would have to pay to B £100, which is the *difference* between the value of £10000 at 99 per cent., and the value of £10000 at 98 per cent.; for A is *supposed* to sell to B for £98 what he is *supposed* to buy for £99.—Again, if this particular stock should fall on the Thursday to 97 per cent., then B would have to pay to A £100, for B is *supposed* to buy of A for £98 what is in reality only worth £97. In order for A and B to effect this transaction, it is not necessary for either of them to have one farthing in that particular stock, or any other stock; so long as they can pay the differences of these imaginary sales their credit is good in the money market.

There are eight account days in the year, one in each of the months, January, February, April, May, July, August, October and November. The January and July accounts are always on the opening of the 3 per cent. consols. The other account days are fixed by the Bankers and Merchants who often lend large sums of money and take stock as security, these account days being for the repayment of the money, and the retransfer of the stock.

*Exchequer-Bills* are bills for different sums issued by Government, when in want of temporary loans, and they bear a daily interest, of from 1½*d*. to 2½*d*. for every £100 so borrowed, according to the state of the money market, the lower rate when money is plentiful, and a *higher rate* when money is scarce.

The interest due upon outstanding Exchequer-Bills is calculated daily to determine their precise value. The *precise* value of an outstanding Exchequer-Bill being its price in "the market," together with the interest due upon such Exchequer-Bill.

The value of these Exchequer-Bills rises and falls with the price of Stock. When we find in the list "Exchequer-Bills 16*s*. 18*s*. pm.," it signifies that every £100 in Exchequer-Bills can be sold by the public for £100 16*s*. or be bought by them at £100 18*s*. Persons having large balances, or bankers who may be required to make large payments at a short notice, usually invest their money in these securities.

The amount of Exchequer-Bills usually in circulation varies, being now (1842) about 18½ millions; during the late war it was much larger, having been in 1815 upwards of 67 millions.

Examples in this Rule are worked by the Rule of Three.

## EXAMPLES.

1. What must be paid for £970 in the 3 per cent. consols at 89¾ per cent., brokerage £⅛ per cent.

```
      £     £      £                          £
As  100 : 970 :: 89⅞    : purchase money of 970 stk.
    100 : 970 :: 89·875 :
                    970
                 -------
   89¾            6291250
    ⅛            808875
  -----          --------
   89⅞     100)  87178·750
                 ---------
                  871·7875
8) 7·000                20
  ------          --------
   ·875            15·7500
                       12
                     ----
                     9·00
                     ====
```

*Answer*. £871 15*s*. 9*d*.

It will be found very advantageous to use decimals where either of the terms is a compound quantity or mixed number, as in the preceding example. If the Simple Rule of Three had been used, it must have been stated, thus.

```
      £       £          £  s.  d.
As  100  :  970  ::     89  17   6  :  purchase money.
```

$£89\frac{7}{8}$ being equal to £89 17*s.* 6*d.*

2. What amount of stock in the $3\frac{1}{2}$ per cents. at $99\frac{3}{8}$ will £5000 purchase, brokerage $£\frac{1}{8}$ per cent. on sum purchased?

$$\begin{array}{lcccccc} & £ & & £ & & £ & \\ \text{As } 99\tfrac{3}{8}+\tfrac{1}{8}=99\tfrac{1}{2} & : & 5000 & :: & 100 & : & \text{stock required.} \\ 99{\cdot}5 & : & 5000 & :: & 100 & : & \text{stock required.} \end{array}$$

```
                        100
                  ─────────────
          99·5)   500000·0000  (5025·1256
                  4975                 20
                  ────               ──────
                   2500              2·5120
                   1990                  12
                   ────              ──────
                    5100              6·144
                    4975
                    ────
                     1250
                      995
                     ────
                      2550
                      1990
                      ────
                       5600
                       4975
                       ────
                        6250
                        5970
                        ────
                         280
                         ═══
```

*Answer.* £5025 2*s.* 6*d.*

3. If a person purchases stock in the 3 per cent.

Red. at 89¾, brokerage ⅛ per cent. What per cent. does he get for his money ?

```
      £   £   £       £     £                £
As 89¾ + ⅛ = 89⅞ : 100 :: 3 : interest on 100 cap.
             89·875 : 100 :: 3 : interest on 100 cap.
                      3                     £
                   ──────────────────
          89·875) 300·000000000 (3 337969
                  269625                 20
                  ──────             ────────
                   303750            6·759380
                   269625                  12
                   ──────             ───────
                    341250            9·11256
                    269625
                    ──────
                     716250
                     629125
                     ──────
                      871250
                      808875
                      ──────
                       623750
                       539250
                       ──────
                        845000
                        808875
                        ──────
                         36125
                         ═════
```

*Answer*. £3 6*s*. 9*d*.

4. When the 3 per cent. consols are at 88½, what must be paid for £1735 stock, including brokerage ⅛ per cent. ?

5. What amount of stock in the 3½ per cent. Red. at 99⅛ will £3500 purchase, brokerage ⅛ per cent. on the sum purchased ?

6. If a person purchases India Stock, paying a dividend of 10½ per cent., at 247½, what per cent. does he get for his money ?

7. If I sell out £4375 stock in the new 3½ per cent. at 98¼, brokerage ⅛ per cent., what do I receive in cash ?

8. A person is desirous of investing a sum of money in the Funds; the 3 per cent. Red. are at 90⅛, the 3½ per. cent. Red. at 99¼, brokerage ⅛ per cent. in both. In which stock does he get the greater interest for his money, and what is the difference?

9. If I have £5000, and buy it into the 3 per cent. Red. at 91¼, brokerage ⅛, how much stock shall I possess; and what will be the amount of the half-yearly dividend payable thereon?

## DISCOUNT.

**Discount** is an allowance of so much per cent., previously agreed upon, made for the payment of any sum of money before it becomes due; or an allowance made by the creditor to the debtor, when he pays *cash* instead of taking the usual *credit*, according to the custom of the trade the creditor exercises.

That an allowance for paying money before it becomes due ought to be made is plain, for if a person keeps the amount of a debt in his own hands till the time it becomes due, he can make an interest on that money, and if he pays it before it becomes due, he gives the advantage of making that interest to the party receiving the debt.

The *discount* subtracted from the sum or debt will give the present worth.

The *present worth* of any sum or debt is such a sum as would, if put out to interest for the time and at the rate at which the discount is made, amount to that sum or debt.

To find the discount of any sum or debt.

**Rule.** As the amount of £100 for the given time and at the given rate : the interest of £100 for the given time and at the given rate :: the sum or debt, which is the amount of the present worth for the given time and at the given rate : to the discount.

To find the present worth of any sum or debt.

RULE. As the amount of £100 for the given time and at the given rate : £100 :: the sum or debt, which is the amount of the present worth for the given time and at the given rate : to the present worth.

This rule is chiefly applicable to Bill Transactions. *Bills* are drafts, or promissory notes, by which a person engages to pay a sum of money at a certain time hereafter, therein named, this sum being the price of goods bought, &c. The following is a form of one of these bills.

£275 15*s*. *London, Sep.* 28, 1841.

*Three months after date, pay to Mr. Thomas Smith, or order, the sum of two hundred and seventy-five pounds, fifteen shillings, value received.*

*James Brown.*

*To Mr. R. Jones,*
*Cheapside, London.*

Here James Brown is the *drawer* of the Bill, and Richard Jones is the *acceptor*, who writes his name and place where the Bill is payable across the face of the Bill, and is the person on whom the Bill is drawn, and who pays the Bill when due; if Thomas Smith, the party holding the Bill, pays it away in the course of business, he writes his name across the back of the Bill, by which he renders the same negociable (able to be passed as money in the course of business).

EXAMPLES.

1. What is the discount and present worth of £100 *for 1 year* at 5 per cent.?

£
5 = interest of £100 for 1 year
100
105 = amount of £100 for 1 year at 5 per ct.

As £105 : £5 :: £100 : discount required.
100
105) 500 (4
420
80
20
105) 1600 (15
105
550
525
25
12
105) 300 (2
210
$\frac{90}{105} = \frac{6}{7}$.

| £ | s. | d. | |
|---|---|---|---|
| 100 | 0 | 0 | sum due |
| 4 | 15 | 2$\frac{6}{7}$ | discount |
| £95 | 4 | 9$\frac{1}{7}$ | present worth. |

EXPLANATION. In the preceding example as £105, which is £100 together with the interest of £100 for 1 year at 5 per cent. : the interest of £100 for 1 year at 5 per cent., viz. £5 :: £100 which is the present worth of £100, viz. £95 4*s*. 9$\frac{1}{7}$*d*., together with the interest of that present worth for 1 year at £5 per cent., viz. £4 15*s*. 2$\frac{6}{7}$*d*. : discount, viz. £4 15*s*. 2$\frac{6}{7}$*d*. or interest of the present worth of £100 for 1 year at 5 per cent. That the present worth of £100, viz. £95 4*s*. 9$\frac{1}{7}$*d*. put to interest for 1 year at 5 per cent. will amount to £100 is easily shown, thus.

$$\begin{array}{lcrrrcrcl} & \pounds & & \pounds & s. & d. & & \pounds & \\ \text{As} & 100 & : & 95 & 4 & 9\frac{1}{7} & :: & 5 & : \text{int. required} \\ & & & & & 5 & & & \\ \hline & & & 4{,}76 & 3 & 9\frac{5}{7} & & & \\ & & & 20 & & & & & \\ \hline & & & 15{,}23 & & & & & \\ & & & 12 & & & & & \\ \hline & & & 2{,}85\frac{5}{7} & = & \dfrac{85\frac{5}{7}}{100} & = & \frac{600}{700} = \frac{6}{7}. & \end{array}$$

$$\begin{array}{rrrl} \pounds & s. & d. & \\ 95 & 4 & 9\frac{1}{7} & \\ 4 & 15 & 2\frac{6}{7} & \\ \hline \pounds 100 & 0 & 0 & \text{Amount.} \end{array}$$

By a similar process it will be seen that the discount, viz. £4 15*s*. 2$\frac{6}{7}$*d*. put out to interest for 1 year at 5 per cent. will amount to £5, the interest of £100 for 1 year at 5 per cent.

Thus the creditor receives a sum which if put out to interest for the given time and rate, would amount to the debt; and the debtor has a sum allowed to him which if put out to interest for the given time and rate would amount to the interest of the debt for that given time and rate, so that neither party has an advantage over the other.

2. What is the present worth of £735 16*s*. payable 4 months hence, at 4 per cent.?

3. What is the discount of £495 3*s*. 9*d*. due 7 months hence, at 5 per cent.?

4. What is the present worth of £97 13*s*., due 9 months hence, at 3$\frac{1}{2}$ per cent.?

5. Find the discount of a bill for £373, having 2 months and 15 days to run, at 4 per cent.

6. What is the present worth of £250 18*s*. 9*d*. due 15 months hence, at 4$\frac{1}{2}$ per cent.?

# BARTER.

**Barter** is the exchanging one commodity for another. The rule called Barter teaches tradesmen so to exchange the commodities in which they deal, that neither party may sustain a loss by that exchange.

**Rule.** Find, by the Rule of Three, what quantity of the commodity whose *price only* is given, may be bought for the *value* of the commodity *whose price and quantity* are given.

## Examples.

1. A, a hop grower, has hops at £7 10*s.* per cwt., which he wishes to exchange with B, a farmer, for 100 quarters of oats at 23*s.* 6*d.* per quarter. How many cwts. of hops must A give to B ?

100 quarters at 23*s.* 6*d.* per qr.=£117 10*s.*

|  | £ | *s.* |  | £ | *s.* |  | *cwt.* |  |  |
|---|---|---|---|---|---|---|---|---|---|
| As | 7 | 10 | : | 117 | 10 | :: | 1 | : | cwts. of hops required |
| or as | 7·5 |  | : | 117·5 |  | :: | 1 | : | cwts. of hops required |
|  |  |  |  | 1 |  |  |  |  | cwts. |

|  |  |
|---|---|
| 7·5) 117·50000 ( | 15·6666 |
| 75 | 4 |
| 425 | 2·6664 |
| 375 | 28 |
| 500 | 53312 |
| 450 | 13328 |
| 500 | 18·6592 |
| 450 |  |
| 50 &c. |  |

*Answer.* 15 cwt. 2 qrs. 18·6592 lbs.

2. A tallow chandler is desirous of exchanging candles at 7*s*. per dozen lbs. with a grocer for 3 cwt. 2 qrs. 17 lbs. of sugar at 11*d*. per lb. How many candles must he give for that quantity of sugar?

3. A farmer exchanges wheat at 63*s*. per quarter with a butcher for 7 cwt. 3 qrs. 14 lbs. of pork at 6½*d*. per lb. What quantity of wheat must the butcher receive?

4. How much tea at 4*s*. 9*d*. per lb. must be given in exchange for 1 cwt. 1 qr. 14 lbs. of sugar at £3 7*s*. 6*d*. per cwt.?

5. A wishes to exchange 6 hogsheads of brandy at 32*s*. per gallon with B for port wine at 13*s*. 9*d*. per gallon. What quantity of wine must A receive?

6. A clothier exchanges 137 yards of broad cloth at 9*s*. 4*d*. per yard with an oilman for £25 in money, and the remainder in oil at 5*s*. 3*d*. per gallon. How much oil must the clothier receive?

7. A grazier at Stilton, exchanges 159 cheeses, weighing on the average 13¼ lbs. each, at 1*s*. 1½*d*. per lb., with a grocer for £80 in money and the remainder in sugar at 10½ per lb. How much sugar must the grazier receive?

## PROFIT AND LOSS.

PROFIT AND LOSS is a rule which discovers what is gained or lost in the buying or selling of goods, and teaches merchants and tradesmen so to raise or lower the prime cost of their goods, that they may gain or lose so much per cent.

The *prime cost*, or first cost, is the price paid to the manufacturer, grower, or importer of any commodity by the merchant or tradesman, on which price he is to make his profit.

Questions in this rule are nothing more than com*plicated* questions in the Rule of Three.

## EXAMPLES.

1. If I buy coffee at 1*s*. 6*d*. per pound, and sell it for 2*s*. per pound, what do I gain per cent.?

| *s.* | *d.* | |
|---|---|---|
| 2 | 0 | selling price |
| 1 | 6 | prime cost |
| 0 | 6 | gain upon the prime cost |

| | *s. d.* | | £ | | | | |
|---|---|---|---|---|---|---|---|
| As | 1 6 | : | 100 | :: | 6*d.* | : | gain upon £100. |
| | 12 | | 20 | | | | |
| | 18 | | 2000 | | | | |
| | | | 12 | | | | |
| | | | 24000 | | | | |
| | | | 6 | | | | |

18) 144000

12) 8000

2,0) 66,6 8

33 6 8

*Answer.* £33 6*s*. 8*d*.

2. If I buy sugar at 4 guineas per cwt., and sell it at 11½*d*. per lb., whether do I gain or lose per cent., and how much?

| *lbs.* | | *d.* | | £ | *s.* | *d.* | |
|---|---|---|---|---|---|---|---|
| 112 | at | 11½ | per lb.= | 5 | 7 | 4 | selling price |
| | | | | 4 | 4 | 0 | prime cost |
| | | | | 1 | 3 | 4 | gain upon the prime cst. |

| | £ *s.* | | £ | | £ *s.* *d.* | | |
|---|---|---|---|---|---|---|---|
| As | 4 4 | : | 100 | :: | 1 3 4 | : | gain upon £100 |
| or as | 4·2 | : | 100 | :: | 1·166666 | : | gain upon £100 |

100 £

4·2) 116·666666 (27·77777
84 20

326 15·55540
294 12

326 6·6648
294 4

326 &c. 2·6592

*Answer.* I gain £27 15*s.* 6½*d.* nearly.

3. If I buy tea at £25 4*s.* per cwt., at what price must I sell it per lb. to gain 15 per cent.?

| | *cwt.* | | *lb.* | | £ *s.* | | |
|---|---|---|---|---|---|---|---|
| As | 1 | : | 1 | :: | 25 4 | : | prime cost of 1 lb. |
| | 112 | | | | 20 | | |
| | 112 | | | | | | |

112) 504 (4*s.* 6*d.* prime cost of 1 lb.
448

56
12

112) 672 (6
672

| | £ | | £ | | £ | | £ *s.* *d.* | | |
|---|---|---|---|---|---|---|---|---|---|
| As | 100 | : | 100 + 15 | = | 115 | :: | 4 6 | : | prime cost + 15 per ct. gain |

$$\begin{array}{lrrll} & \pounds & \pounds & \pounds\ \ s. & \\ \text{or as} & 100\ : & 115 & ::\ 4\ \ 5\ : & \left\}\begin{array}{l}\text{prime cost} +\\ \text{15 per ct. gain}\end{array}\right. \end{array}$$

$$\begin{array}{r} 115\\ 4{\cdot}5\\ \hline 575\\ 460\phantom{0}\\ \hline 100)\ 517{\cdot}5\\ \hline 5{\cdot}175\\ 12\\ \hline 2{\cdot}100\\ \hline\hline \end{array}$$

*Answer.* 5*s.* $2\frac{1}{10}$*d.*

4. If I buy tea at 3*s.* 9*d.* per lb. and sell it at 4*s.* 7*d.* per lb., what do I gain per cent.?

5. Bought sugar at 7*d.* per lb., which I sell at £3 10*s.* per cwt., whether do I gain or lose per cent., and how much?

6. If I buy goods at 12 guineas per cwt., how must I sell them per lb., to gain 15 per cent.?

7. If I gain 1*s.* 7½*d.* in the £, what is the gain per cent.?

8. Bought tea at 4*s.* 2*d.* per lb., but, the markets falling I am constrained to sell it at a loss of 12 per cent.: at what price must I sell my tea per lb.?

9. A wine merchant in London buys 35 hogsheads of sherry at Cadiz for 9*s.* 3*d.* per gallon, on its carriage home one hogshead is staved and the wine entirely lost, the contents of another hogshead tasting of the cask, he is obliged to part with it for half its cost price; at what rate must he sell the remainder so as to gain 20 per cent. by the whole?

10. If, when a hop merchant sells part of a consignment of hops at £7 10*s.* per cwt., he gains 10 per cent.; what will he gain per cent. when he sells the remaining part of the same hops for 10 guineas per cwt.?

11. Bought 17 pieces of silk each containing 35 yards for £123 18*s.* 4*d.*, but they not proving so good

as I had expected, I sell them at a loss of 7 per cent. At what do I sell them per yard?

## FELLOWSHIP.

**Fellowship** teaches merchants, &c., trading with a joint stock, to assign to every partner his particular share of the gain or loss, according to the money he has advanced.

Persons trading with a joint stock, or in company, are said to be in *partnership* with each other. The partners collectively are called *the Firm.*

Fellowship is of *two* kinds, Single and Double.

*Single Fellowship* is where the same or different stocks are employed for any the *same* time.

*Double Fellowship* is where the same or different stocks are employed for *different* times.

## SINGLE FELLOWSHIP.

**Rule.** As the amount of the various stocks : each man's particular stock :: the whole gain or loss : each man's particular gain or loss.

By this rule a bankrupt's estate may be divided among his creditors, and legacies may be adjusted, when more has been devised than the estate can pay.

### Examples.

1. Three persons, denoted by A, B, and C, are in partnership, A puts in £300; B £400; C £500; on making up their books at the end of the year, they find that they have gained £350; what is each man's share of the gain?

£300 + £400 + £500 = £1200

£ £ £
As 1200 : 300 :: 350 : A's share of the gain
350

12,00) 1050,00

£87 10*s.* A's share.

£ £ £
As 1200 : 400 :: 350 : B's share of the gain
350

12,00) 1400,00

£116 13*s.* 4*d.* B's share.

£ £ £
As 1200 : 500 :: 350 : C's share of the gain
350

12,00) 1750,00

£145 16*s.* 8*d.* C's share.

2. A and B gain by trading £72, A put into the stock £120, and B £150; what is each man's share of the profits?

3. Three persons enter into partnership, A puts in £327; B £413; and C £550; they gain £735. What is each man's share of the profits?

4. Three merchants enter upon a speculation, A puts in £320; B £230; C £180; they gain £1375 11*s.* 10*d.* What is the gain of each?

5. A gentleman dies, leaving his property among four persons, whose shares were to be as follows. A £4000 ; B 3500 ; C £5000 ; and D £1500. His executors, after paying his debts and funeral expences, &c., find that they have only £12750 remaining. What sum must they pay to each of the legatees?

6. A tradesman fails in business owing to A £314 17*s.*; to B £436 11*s.*; and to C £173 15*s.* His assets are £375 17*s.* 6*d.* What will each of his creditors receive?

## DOUBLE FELLOWSHIP.

RULE. Multiply each man's stock by the time of its continuance, and add these products together; then as the sum of the products : each man's particular product :: whole gain or loss : to each man's particular gain or loss.

### EXAMPLES.

1. Two merchants A and B enter into partnership. A puts in £700 for 2 years; B £900 for 3 years. They gain £1600. What is the share of each?

£ £
$700 \times 2 = 1400$ product of A's stock and time

$900 \times 3 = 2700$ product of B's stock and time

£4100

£ £ £
As 4100 : 1400 :: 1600 : A's share of the gain
1600

840000
1400

41,00) 22400,00

£546 6 $9\frac{39}{41}$ A's share.

£ £ £
As 4100 : 2700 :: 1600 : B's share of the gain
1600

1620000
2700

41,00) 4320000

£1053 13 $2\frac{2}{41}$ B's share.

2. A, B, and C, three butchers, hold a piece of meadow land, measuring 7 acres, for which they agree to pay at the rate of £5 per acre, per annum. A puts in 9 oxen for 5 months; B 4 oxen for 8 months; and C 7 oxen for 9 months. What portion of the rent must each pay?

3. Two merchants enter into partnership for 3 years, each contributing £500 to the joint stock. A at the end of 16 months being in want of money draws out £230; and B at the same time puts in the same sum; they gain £643 15*s*. What is each man's just share of the profits?

# INVOLUTION AND EVOLUTION.

INVOLUTION is the method of raising numbers to any required power, and is performed by multiplying a number into itself as many times as there are units in the *index* denoting the power; thus $3^4$ means that 3 is to be multiplied into itself 4 times, or that $3^4=3\times3\times3\times3=81$: where the figure 4 placed to the right hand above the figure 3 is called the *index of the power*.

$3^2=3\times3=9$ is called the square of 3.
$3^3=3\times3\times3=27$ is called the cube of 3.
$3^4=3\times3\times3\times3=81$ is called the fourth power of 3,

&c., &c., &c.

EVOLUTION is the reverse of involution, being the method of extracting the root of any number.

3 is called the square root of 9, for $3\times3=9$.
3 is called the cube root of 27, for $3\times3\times3=27$.
3 is called the fourth root of 81, for $3\times3\times3\times3=81$,

&c., &c., &c.

When it is required to find the root of any number, as the square root of 9, that number is written thus $\sqrt[2]{9}$ where the figure 2 is called the *index of the root*, and $\surd$ the *radical sign*. Radical means *belonging to the root*.

It is only intended here to consider the extraction of the square root, the rule for which must be an effort of memory, as the principle upon which it depends can only be shewn by means of Algebra.

# SQUARE ROOT.

**Square Root** is the method of finding a number, which multiplied by itself will give the number of which the square root is to be found.

Square Root may be divided into four cases.

**Case 1.** To find the square root of a whole number.

**Rule.** Point off the number by placing points or dots over every alternate figure beginning with the unit's place; find the nearest root of the first period, set it to the right hand of the number whose root is to be extracted (after the manner of a quotient); square the root so found, and subtract its square from the first period; to the remainder annex the two figures contained in the second period (after the manner of a dividend), and to the left hand of this number (after the manner of a divisor), place twice the figure in the root, find how often this product is contained in the dividend exclusive of the figure in the unit's place, annex the figure so found to the right hand both of the divisor and the root: multiply the divisor thus found by the figure placed in the root, subtract the product as before, and to the remainder annex the two figures contained in the next period: multiply the figures in the root by two, and use the product for a divisor as before directed, and so on through all the periods.

Should there be a remainder, decimal places may be added to the right hand of the number whose root is to be extracted, and the operation carried on to any required extent, observing to add two ciphers for every figure placed in the root.

The number of figures in the root will be equal to the number of points or periods: and the number of decimal places in the root will be equal to the number of points or periods in the decimal places of the number whose root is to be extracted.

CASE 2. To find the square root of a mixed number composed of a whole number, and a decimal fraction.

RULE. If the number of decimal places be *odd* make them *even* by adding a cipher to the right hand, point off, and proceed as before directed.

OBSERVATION. The number of places of decimals is made even, in order that the first point of the whole number may be over the unit's place.

CASE 3. To find the square root of a vulgar fraction whose numerator and denominator are *not* both square numbers.

RULE. Reduce the vulgar fraction to a decimal fraction, and proceed as before.

CASE 4. To find the square root of a vulgar fraction whose numerator and denominator *are* both square numbers.

RULE. Extract the square root of both the numerator and denominator separately, as before directed. The square root of the numerator placed over the square root of the denominator will be the square root of the fraction required.

## EXAMPLES.

1. Extract the square root of 141376.

$$\begin{array}{r@{\,}l}
 & 1\dot{4}1\dot{3}7\dot{6}\ (376 \quad \text{root} \\
 & \phantom{1}9 \\ \cline{2-2}
67) & \phantom{1}513 \\
 & \phantom{1}469 \\ \cline{2-2}
746) & \phantom{1}4476 \\
 & \phantom{1}4476 \\ \cline{2-2}
\end{array}$$

*Answer.* 376.

2. Extract the square root of 12·2 or $12\frac{1}{5}$.

$1\dot{2}\cdot2\dot{0}0\dot{0}0\dot{0}$ (3·492 +
9

64) 320
256

689) 6400
6201

6982) 19900
13964

5936

*Answer.* 3·492 +

3. Extract the square root of $\frac{2}{7}$.

7) 2·00000000000
·28571428571 &c.

$\cdot2\dot{8}5\dot{7}1\dot{4}2\dot{8}5\dot{7}1\dot{4}$ (·534522 +
25

103) 357
309

1064) 4814
4256

10685) 55828
53425

106902) 240357
213804

1069042) 2655314
2138084

517230

*Answer.* ·534522 +

4· Extract the square root of $\frac{9}{64}$.

$$\begin{array}{ll} 9\ (3 & \qquad 64\ (8 \\ \underline{\underline{9}} & \qquad \underline{\underline{64}} \end{array}$$

therefore $\sqrt[2]{\frac{9}{64}}=\frac{3}{8}$ *Answer.*

5. Extract the square root of 2891105361.
6. Extract the square root of 735·1983.
7. Extract the square root of 15.
8. Extract the square root of $\frac{3}{4}$.
9. Extract the square root of 6431·27315.
10. Extract the square root of ·073.
11. Extract the square root of $\frac{7}{3}$.
12. Extract the square root of $\frac{676}{841}$.
13. Extract the square root of 152415787501905 21.
14. Extract the square root of 2.
15. Extract the square root of 387420489.
16. Extract the square root of 51751139712 10225.

## DUODECIMALS or CROSS MULTIPLICATION.

DUODECIMALS are properly speaking fractions, whose denominators are 12 or its powers.

The rule called *Duodecimals or Cross Multiplication* teaches workmen to take the content of their work in square and cubic feet, &c., and is applicable to rectangular figures *only*, as squares □, oblongs ▭, cubes or solid squares ⧉, &c..

This rule, although easy in *Practice*, involves difficulties: 1st, with regard to its not being a pure Arithmetical operation, but an application of Arithmetic to Geometry. 2nd, with regard to the terms primes, seconds, &c., having *different* values according as they are applied to *superficial* or *solid* measures:

A prime in superficial measure being $\frac{1}{12}$ of a square foot, or 12 *square* inches: and in solid or cubic measure $\frac{1}{12}$ of a cubic foot, or 144 *cubic* inches.

A second in superficial measure being $\frac{1}{12}$ of a prime, or 1 *square* inch; and in solid or cubic measure $\frac{1}{12}$ of a solid prime or 12 *cubic* inches, &c., &c.

This rule may be divided into *two* cases.

1. Where the content of a rectangular *area* is to be found.
2. Where the content of a rectangular *solid* is to be found.

CASE 1. An *area* has length and breadth *only*: the content of any rectangular area is found by multiplying its length by its breadth. The result will be in square feet, &c.

The dimensions made use of are *feet*, *primes*, *seconds*, &c., each successive denomination being $\frac{1}{12}$ of the preceding, 1 square foot being the integer.

When the linear dimensions contain yards, the yards *must* be reduced to feet, in order that the denominations may decrease in a twelvefold proportion from the place of feet towards the right hand, which would not be the case if the denomination yards was retained.

A *square foot* is 12 inches long, and 12 inches broad, and contains 144 square inches.

A *prime* (1′) is 12 seconds or $\frac{1}{12}$ of a square foot.

A *second* (1″) or *square inch* is $\frac{1}{12}$ of a prime or $\frac{1}{144}$ of a square foot.

Hence the following table.

12 seconds or square inches make 1 prime, written 1′
12 primes make 1 sq. foot, written 1f.

When hereafter the expressions feet multiplied by feet, inches multiplied by inches, &c., are used: it is to be understood that numbers *denoting* feet in length, inches in length, &c., are to be multiplied by numbers *denoting* feet in breadth, inches in breadth, &c.

When feet are multiplied by feet, the result will be square feet.

When feet are multiplied by inches, the result will be primes for

3 ft. × 7 in. = 3 ft. × $\frac{7}{12}$ ft. (a linear inch being $\frac{1}{12}$ of a linear foot.)
. . . . . . . . = $\frac{21}{12}$ of a square foot.
. . . . . . . . = 21 primes.
. . . . . . . . = 1 ft. .. $9'$.

When inches are multiplied by inches the result will be seconds or square inches for

3 in. × 7 in. = $\frac{3}{12}$ ft. × $\frac{7}{12}$ ft.
. . . . . . . . = $\frac{21}{144}$ of a square foot.
. . . . . . . . = 21 seconds or square inches.
. . . . . . . . = $1'$ .. $9''$.

RULE. Place the multiplier under the multiplicand so that feet may be under feet, inches under inches. Multiply each denomination of the multiplicand by the number of feet in the multiplier, beginning with the lowest denomination: then multiply each denomination of the multiplicand by the number of inches in the multiplier, and so on, if there be any denomination lower than inches.

In this operation primes must be reduced to square feet, the square feet added to the next product, and the remaining primes set down under the inches in the multiplier: seconds or square inches must be reduced to primes, the primes added to the next product, and the remaining seconds or square inches set down to the right hand of the primes.

Different works are computed by different measures.

Glazing by the foot.

Painting, plumber's work, plastering, paving, &c. by the yard.

Flooring, roofing, tiling, by the square of 100 feet.

Brickwork by the rod of 16½ feet, which is 272¼ or 272·25, square feet.

Bricklayers always value their work at the rate of a brick and a half thick; if the work be more or less thick, it must be reduced to that thickness, by multiplying the area by the number of half bricks in the thickness, and dividing that product by 3.

CASE 2. A *solid* has length, breadth, and thickness or depth: the content of any rectangular solid is found by multiplying its length by its breadth, and that product by its thickness or depth. The result will be in cubic feet, &c.

The dimensions made use of are *solid or cubic feet, solid primes, solid seconds, solid thirds*, &c., 1 solid or cubic foot being the integer. These solid primes, &c. bear the same relative value to the integer, 1 solid or cubic foot, that the primes, &c. bore to the integer, 1 square foot, in Case 1.

A *solid or cubic foot* is 12 inches long, 12 inches broad, and 12 inches thick or deep, and contains 1728 solid or cubic inches, ($12 \times 12 \times 12 = 1728$).

A *solid prime* ($1'$) is 144 solid or cubic inches, or $\frac{1}{12}$ of a solid or cubic foot.

A *solid second* ($1''$) is 12 solid or cubic inches, or $\frac{1}{12}$ of a solid prime, or $\frac{1}{144}$ of a solid or cubic foot.

A *solid third* ($1'''$) or 1 *solid or cubic inch* is $\frac{1}{12}$ of a solid second, or $\frac{1}{1728}$ of a solid or cubic foot.

Hence the following table.

| | | written |
|---|---|---|
| 12 solid thirds or cubic in. make | 1 solid second | $1''$ |
| 12 solid seconds make | 1 solid prime | $1'$ |
| 12 solid primes make | 1 solid or cubic foot, | 1 c.f. |

When square feet are multiplied by linear feet, the result will be cubic feet.

When primes are multiplied by linear feet, the result will be solid primes, for

$$\begin{aligned} 3' \times 7 \text{ ft.} &= \tfrac{3}{12} \text{ sq. ft.} \times 7 \text{ ft.} \\ &= \tfrac{21}{12} \text{ of a cubic foot.} \\ &= 21 \text{ solid primes.} \\ &= 1 \text{ c.f. .. } 9'. \end{aligned}$$

When seconds are multiplied by linear feet the result will be solid seconds, for

$$\begin{aligned}3'' \times 7\text{ ft.} &= \tfrac{3}{144}\text{ sq. ft.} \times 7\text{ ft.}\\ &= \tfrac{21}{144}\text{ of a cubic foot.}\\ &= 21\text{ solid seconds.}\\ &= 1' \,..\, 9''.\end{aligned}$$

When seconds are multiplied by linear inches, the result will be solid thirds, for

$$\begin{aligned}3'' \times 7\text{ in.} &= \tfrac{3}{144}\text{ sq. ft.} \times \tfrac{7}{12}\text{ft.}\\ &= \tfrac{21}{1728}\text{ of a cubic foot.}\\ &= 21\text{ solid thirds.}\\ &= 1'' \,..\, 9'''.\end{aligned}$$

In the same manner it may be shewn that square feet multiplied by inches give solid primes: and that primes multiplied by linear inches give solid seconds.

Rule. Multiply the length by the breadth as in Case 1, and the resulting product by the thickness or depth; this latter product will be the required content.

The same results will in both cases be obtained, if the decimal parts of a foot are used in the place of inches, &c.

## EXAMPLES.

1. How many square feet are there in a slab of marble 3 ft. 9 in. long by 4 ft. 6 in. broad. And what will be its cost at 6*s*. 3*d*. per square foot?

$$\begin{array}{rrr} \textit{ft.} & \textit{in.} & \\ 3 & 9 & \\ 4 & 6 & \\ \hline 15 & 0 & \\ 1 & 10 & 6 \\ \hline 16 & 10' & 6'' \\ \hline\hline \end{array}$$

| *ft.* | | *s.* | *d.* | |
|---|---|---|---|---|
| 16 10′ 6″ at | | 6 | 3 | per sq. foot |
| | | | 4 | |
| | 1 | 5 | 0 | |
| | | | 4 | |
| | 5 | 0 | 0 | |
| 6′ is ½ | 0 | 3 | 1½ | |
| 4′ is ⅓ | 0 | 2 | 1 | |
| 6″ is ⅛ | 0 | 0 | 3 | |
| | £5 | 5 | 5½ | *Answer.* |

## By Decimals.

3 ft. .. 9 in. = 3·75
4 ft. .. 6 in. = 4·5
1875
1500
16·875
12
10·500
12
6·000

16·875
6*s.* 3*d.* = 6·25
84375
33750
101250
105·46875
12
5·62500
4
2·50000

*s.* *d.*
20) 105 5½ ·5
£5 5 5½ ·5

2. Find the solidity of a rectangular block of marble, whose dimensions are as follows, length 6 ft. 7 in., breadth 4 ft. 8 in., thickness 5 ft. 5 in.: and what will be its cost at 7*s.* 9*d.* per cubic foot?

| *ft.* | *in.* | | |
|---|---|---|---|
| 6 | 7 | | |
| 4 | 8 | | |
| 26 | 4 | | |
| 4 | 4 | 8 | |
| 30 | 8 | 8 | |
| 5 | 5 | | |
| 153 | 7 | 4 | |
| 12 | 9 | 9 | 4 |
| 166 | 5′ | 1″ | 4‴ |

| | | *s.* | *d.* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 7 | 9 | | | | | | |
| | | | 10 | | | | | | |
| | 3 | 17 | 6 | | | | | | |
| | | | 10 | | | | | | |
| | 38 | 15 | 0 | | price of | 100 c.f. | | | |
| | 23 | 5 | 0 | | price of | 60 c.f. | | | |
| | 2 | 6 | 6 | | price of | 6 c.f. | | | |
| 4′ is $\frac{1}{3}$ | 0 | 2 | 7 | | price of | | 4′ | | |
| 1′ is $\frac{1}{4}$ | 0 | 0 | 7$\frac{3}{4}$ | | price of | | 1′ | | |
| 1″ 4‴ is $\frac{1}{9}$ | 0 | 0 | 0$\frac{3}{4}$ | $\frac{4}{9}$ | price of | | | 1″ | 4‴ |
| | £64 | 9 | 9$\frac{1}{2}$ | $\frac{4}{9}$ | price of | 166 | 5′ | 1″ | 4‴ |

3. Multiply 13 ft. 7 in. by 19 ft. 5 in.

4. Multiply 37 ft. 2 in. by 8 ft. 11 in.

5. Multiply 19 ft. 8 in. by 33 ft. 4 in.

6. What will be the cost of glazing a window whose height is 9 ft. 7 in. and breadth 4 ft. 3 in. at 15$\frac{1}{2}$*d.* per foot?

7. The length of a court yard is 47 ft. 5 in., the breadth 35 ft. 3 in. What will the paving come to at 3*s*. 2*d*. per yard?

8. The front of a house, which has 3 tiers of windows, 3 windows in each tier, wants new glazing. The height of the first tier is 7 ft. 3 in., of the second 6 ft. 4 in., of the third 3 ft. 7 in., and the breadth of each window is 4 ft. 3 in. What will the glazier's bill amount to, the charge for glazing being 1*s*. 9*d*. per square foot?

9. What will the wainscoting of a room cost, at 3*s*. 9*d*. per yard, whose height is 11 ft. 3 in., and circumference 87 ft. 6 in.?

10. The ceiling of a room is 20 ft. 7 in. by 18 ft. 4 in.; what will the plastering amount to at 11½*d*. per yard?

11. What will be the price of a brickwall that is 73 ft. 3 in. long, 15 ft. 7 in. high, and 2 bricks thick, at 11 guineas a rod?

12. What will be the cost of papering a room that is 25 ft. 3 in. long, 18 ft. 4 in. broad, and 13 ft. 6 in. high, at 1*s*. 7½*d*. per square yard, deducting the areas of door and windows which are as follows: 2 windows each 4 ft. 9 in. broad, and the same height as the room, and 1 door 10 ft. high and 4 ft. 9 in. broad?

13. If the length of one side of a cube is 2 ft. 3 in., what is its content in cubic feet?

14. Find the price of a rectangular block of stone measuring 17 ft. long, 5 ft. 6 in. broad, and 4 ft. 9 in. high, at the rate of 2*s*. 3*d*. per cubic foot?

15. How many gallons are contained in a rectangular cistern whose dimensions are 28 ft. long, 19 ft. 6 in. broad, and 16 ft. 7 in. deep, there being 277·274 cubic inches in an imperial gallon?

16. How many quarters of corn are there upon a floor which is 18 ft. 9 in. long, and 15 ft. 5 in. broad, the corn being 2 ft. 4 in. deep, 2218·192 cubic inches making one imperial bushel?

17. A gentleman is desirous of making a cistern to contain 12000 imperial gallons, the nature of the ground, where the cistern is to be made, will only allow of its

being 13 ft. broad and 15 ft. 6 in. long. What must be the depth of the proposed cistern?

18. The pedestal of an equestrian statue is 13 ft. 3 in. long, 9 ft. 5 in. broad, and 7 ft. 6 in. high. How many cubic feet of stone does it contain, and what was the cost of the stone at 2*s.* 7½*d.* per foot?

THE END.

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